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AN APPROXIMATE LAW OF FATIGUE IN THE SPEEDS OF RACING ANIMALS.

BY A. E. KENNELLY.

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RACES between swift men, or between swift horses, have been of the greatest interest in all times. Olympia and Epsom Downs are known to fame by the races they have witnessed. Olympian races, recently revived, are of international interest.

It is strange that, judging from encyclopedias and text-books on athletics, there is very little published information concerning the speeds at which races are run. Apparently, all that is known by our books on these matters is that short races are run at higher speeds than long races. Every one knows that a contestant in a mile or kilometer race runs at a lower speed than a sprinter in a 100-yard or 100-meter dash.

There has, however, been accumulated during the last century, and particularly during the last fifty years, a considerable fund of publicly recorded information concerning the record times in which races of stated length have been run. Athletes are, for example, generally familiar with the records of the 100-yard and the mile runs; namely, 9.6 and 252.75 seconds, respectively. A reduction of either of these record times by even one per cent would be a matter of world-wide importance and the hero of the new record would be famous among the inhabitants of the temperate zones.

This paper presents the data which the writer has been able to collect upon record speeds in various kinds of racing, as well as the conclusions that seem to be warranted thereby. It will be seen that the records align themselves closely to a simple mathematical relation. It is not pretended that the records conform rigorously to this mathematical relation. Such a condition could hardly be expected from the performances of different animals at different times and in different parts of the world. It is claimed, however, that any one who will analyze the records presented will be able to satisfy himself that they approximate to the said mathematical relation for practical purposes within satisfactorily small limits of deviation.

We may commence with horse-racing records.

HORSES TROTTING.

Table I, the data of which are taken from page 259 of "The World Almanac and Encyclopedia" for 1906, gives in column I the date of the record, in column II the distance run, or length of the course, and in column III the best record time. For these data "The World Almanac" is made responsible. These data have been checked, however, by those given in the same publication for preceding years. No event has been rejected. The best records for 1, 2, 3, 4, 5, 10, 20, 30, 50, and 100 miles of trotting are taken. They are stated to be World's records, and at least one, — the 4-mile event — is stated to have been made in England.

Commencing with the above data, column IV shows the distances expressed in meters. The meter and kilometer are so much simpler to deal with numerically than the foot, yard, furlong, and mile, that it is worth while to reduce all distances to meters. Column V gives the average speed at which the record was made, expressed in meters per second. Thus, taking the first event, the mile (1609.3 meters) was trotted in 118.5 seconds. This represents an average speed of $1609.3 \div 118.5 = 13.58$ meters per second (30.4 miles per hour; or 44.5 feet per second).

Turning now to Figure 1, the abscissas are laid off both in miles and in kilometers, as far as 20 miles (32.2 kilometers). The ordinates represent speeds both in meters per second and in miles per hour. Another scale of ordinates gives the record time of each run in seconds. It is seen that the speeds, taken from column V, drop from 13.58 meters per second (30.4 miles per hour) at 1 mile (1609 meters) to 9.18 meters per second (20.6 miles per hour) at 20 miles. The average speed of the trotting horse that made the 20-mile record was therefore 67.6 per cent, or about two-thirds of that of the trotting horse which made the 1-mile record.

Taking next the time ordinates, the rising line in Figure 1 closely follows the first six successively increasing times. It is evident that both the speed-distance line and the time-distance line are curves, when thus plotted. The curvature of these curves is greatest near the start, or over the short courses, and diminishes as the course increases.

If, however, the speed and the time with respect to distance be plotted on logarithm paper, as in Figure 2, instead of on ordinary cross-section paper, as in Figure 1, the points fall approximately upon straight lines.

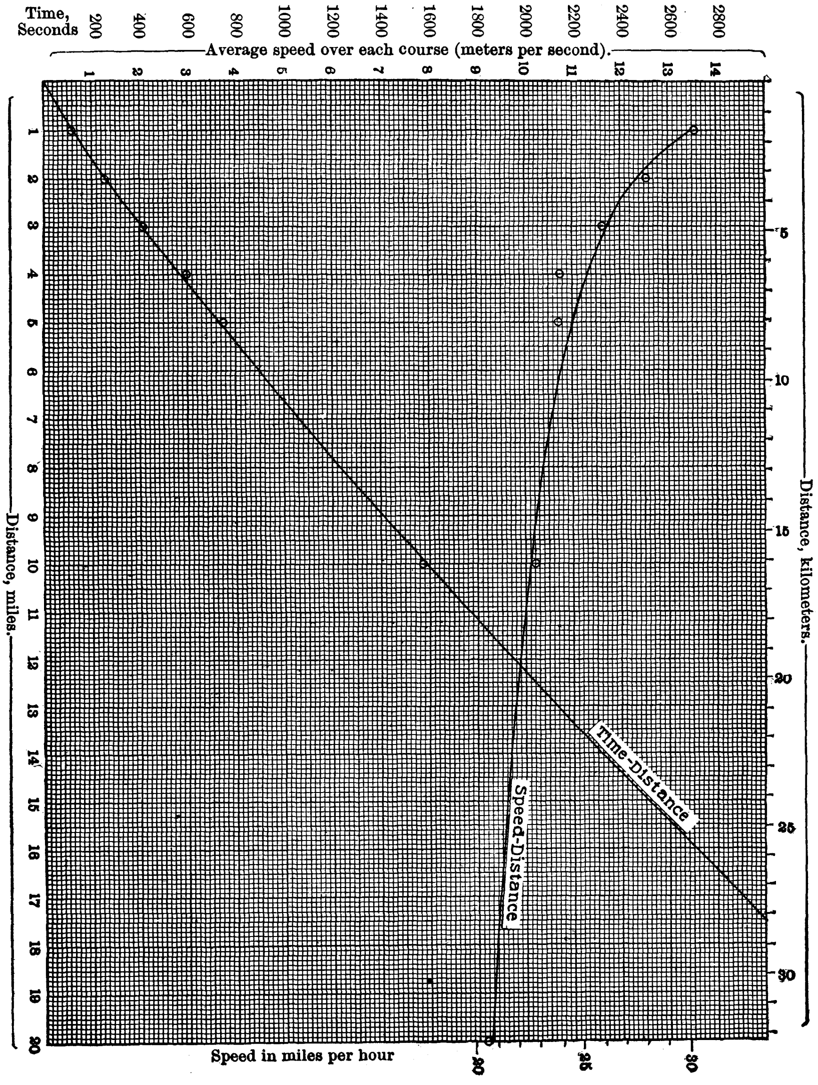
The above fact is the gist of this paper. That is to say, if we consider the three quantities L, T, and V, or length of course, record

TABLE I.

Analysis of World's Trotting Records (Harness Racing).

I. Date.	II. Distance, miles.	III. Time, seconds. T	IV. Distance L, meters.	V. Speed V, $\frac{\text{meters}}{\text{seconds}}$.	VI. log. L.	VII. log. T.	VIII. log. V.	IX. log T' computed.	X. T' computed.	XI. deviation, $T' - T$.	XII. Per cent, $100\left(\frac{T'-T}{T}\right)$.
1905	1	118.5	1,609.3	13.58	3.2066	2.0737	1.1329	2.0774	119.5	1.0	0.8
1902	2	257.0	3,218.7	12.52	3.5077	2.4099	1.0978	2.4162	260.7	3.7	1.4
1893	3	415.5	4,828.0	11.62	3.6838	2.6186	1.0652	2.6143	411.4	–4.1	1.0
1899	4	598.0	6,437.4	10.77	3.8087	2.7767	1.0320	2.7548	568.6	–29.4	4.9 } 1.8
1893	5	750.75	8,046.7	10.72	3.9056	2.8755	1.0301	2.8638	730.8	–20.0	2.8
1893	10	1,575.0	16,094.0	10.22	4.2066	3.1973	1.0093	3.2024	1,594.0	19.0	1.2
1865	20	3,505.0	32,187.0	9.18	4.5077	3.5447	0.9630	3.5414	3,477.0	–28.0	0.8
1857	30	6,479.0	48,280.0	7.45	4.6838	3.8115	0.8723	3.7393	5,484.0	–995.0	15.0
1846	50	14,140.5	80,467.0	5.69	4.9056	4.1504	0.7552	3.9888	9,746.0	–4394.5	31.0
1853	100	32,153.0	160,930.0	5.01	5.2066	4.5072	0.6994	4.3274	21,250.0	–10,900.0	34.0

Figure 1. *World's Trotting Records.*



time, and average speed of the run, in a series of events, any one, say T, plotted against either of the others, say L, the observations are found to fall approximately upon a straight line, with logarithm paper. In other words, the curves plotted on plain rectangular paper from the same records are approximately simple exponential curves, of the type $y = x^n$.

Instead of plotting the quantities in the ordinary way upon logarithm paper to produce straight lines, we may perform the equivalent operation of plotting the logarithms of the quantities upon ordinary scaled paper, and produce similar straight lines. That is, we may plot any one of the quantities $\log L$, $\log T$, and $\log V$ against either of the other two. For some purposes the latter method is to be preferred, although it takes more time. Its application is presented in Figure 3, where $\log T$ and $\log V$ are both plotted as ordinates against $\log L$ as abscissas for all of the data of Table I. Columns VI, VII, and VIII in the table contain the common logarithms of the entries in columns IV, III, and V respectively. It is seen in Figure 3 that the speeds fall closely upon the descending straight line, as far as the 20-mile distance, as already seen in Figure 2. Beyond the 20-mile distance, the speeds fall off markedly and are much too low to meet the line. Table I indicates, however, that these long-distance records of 30, 50, and 100 miles respectively, were made about 50 years ago, whereas the short-distance records are of recent date. At the dates indicated (1846, 1853, 1857) the short-distance trotting records were by no means so good as they are to-day. It is reasonable to assume that if these deviating long distances were attempted to-day, their records would be materially improved.[1]

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[1] Since this paper was written, the writer has been indebted to Prof. E. L. Mark for a photographic curve-sheet pertaining to a paper presented by Prof. Francis E. Nipher to the St. Louis meeting in 1903 of the American Association for the Advancement of Science. The curve-sheet shows the steady reduction in the record times of the trotting-horse mile and also of the running-horse mile at different dates between 1840 and 1903. The curves indicate a final limit to the trotting mile at 98 seconds and a final limit to the running mile at 91.5 seconds. The equations to the curves do not appear on the sheet, but have been computed by the writer, from the curves, as follows: —

At any epoch y years after 1840, the trotting-horse mile record approximates to

$$T_y = 98\,(1 + 0.56 \times 10^{-0.00526\,y}) \quad \text{seconds} \qquad \text{(a)}$$

and for the running-horse mile record:

$$T_z = 91.5\,(1 + 0.154 \times 10^{-0.00993\,z}) \quad \text{seconds} \qquad \text{(b)}$$

where z is the epoch in years after A.D. 1863.

The ascending line in Figure 3, connecting log T and log L, represents the sequence of record times with satisfactory precision, as far as 20 miles, with the exception, perhaps, of the 4-mile event, in which the time is long and the speed low. As shown in Figure 1 or Table I, the speed in the 4-mile trot is only half of one per cent greater than the speed in the 5-mile trot. It should be relatively faster by more than this amount, to judge by the speeds in the other events, and this means that it should lie nearer the straight line of time-distance in Figure 3.

Beyond 20 miles, the points deviate markedly from the rising straight line of Figure 3 in the direction of excessive time, or low speed, as already considered.

The rising straight line of Figure 3 represents the equation

$$\log T = \tfrac{9}{8} \log L - 1.53 \qquad (1)$$

while the falling straight line of speed-distance corresponds to

$$\log V = 1.53 - \tfrac{1}{8} \log L. \qquad (2)$$

That is, the rising line makes with the axis of abscissas an angle of 48° 22′, whose tangent is $\frac{9}{8}$; while the falling line makes with the same axis an angle of $-7° \, 7' \, 30''$, whose tangent is $-\frac{1}{8}$.

Equation (2) implies that at $L = 1$, or upon a course one meter long, the speed of trotting would be 33.9 meters per second, the logarithm of this number being 1.53. It would be impossible for a horse to reach any such speed on such a very short course, even with flying start, if only owing to inertia and the large effort required for initial acceleration. The initial velocity of equation (2) is therefore a fictitious quantity of merely theoretical interest. The speed curve of Figure 1 and the straight speed lines of Figures 2 and 3 mark a satisfactory application of equation (2) between the limits of 1 mile and 20 miles. Columns IX and X of Table I give the computed time for each event, as determined by equation (1) or the rising lines in Figures 1, 2, and 3. Column XI gives the deviation or discrepancy between the

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From formula (a) or from the curve-sheet, the ratios of reduction in the trotting-horse mile record time to 1905 are,

at 1846	0.82
" 1853	0.844
" 1857	0.858

Using these correcting ratios, the 30-mile trotting record is brought on to the logarithmic straight line in Figure 3; while the 50-mile and 100-mile records are only brought about half-way towards that line, as indicated on the figure.

computed record time T' seconds and the published record time T seconds, while column XII expresses this discrepancy in percentage of the record time T. Thus, the 4-mile event should have been trotted in 568.6 seconds by the formula, as against the published record of 598 seconds, a discrepancy of 29.4 seconds or 4.9 per cent of the published record. It is seen that between the limits of 1 mile and 20 miles (1.61 and 32.2 kilometers) the average discrepancy between the recorded time and the time taken from the equation (1) or the ascending lines in Figures 2 and 3 is 1.8 per cent. The discrepancy is much greater in the three longest events and reaches 34 per cent in the 100-mile trot. Owing to the age of these three records, however, it is submitted that they may properly be set aside. At all events, between the limits of 1 mile and 20 miles the straight logarithmic line of times agrees with the published records to an average of 1.8 per cent. If the suspected 4-mile record were set aside, the average discrepancy without regard to sign would come down to 1.1 per cent.

In Figure 4, drawn to uniform scale, the average speeds are continued to 100 miles of course-length, or beyond the limits of Figure 1. The curve of speeds corresponds to equation (2) or to

$$V = \frac{33.9}{L^{\frac{1}{8}}} \text{ meters per second.} \tag{3}$$

The figure shows the discontinuity which exists between the speeds over the three longest courses and those over courses up to 20 miles, as taken from column V, Table I.

HORSES RUNNING.

The records for running-horse races appear in columns I and II of Table II. They are taken from page 258 of "The World Almanac" for 1905, which gives the records for 33 courses between $\frac{1}{4}$ mile (402.3 meters) and 4 miles (6437 meters) on American turf, revised to December 1, 1904. Column III gives the distances in meters and column IV the average speed of each run. The times and the speeds are plotted against distance to uniform scale in Figure 5 and to logarithmic scale in Figure 6. In the latter case the entries in columns V, VI, and VII are used. It is to be noticed that in Figure 5, with uniform ruling, the observations follow curves, whereas in Figure 6, with logarithmic ruling (or logarithms of the quantities on uniform ruling), the observations fall substantially on straight lines. The two curves drawn in Figure 5 respectively correspond mathematically to the two straight lines drawn to meet the observations in Figure 6.

TABLE II.

HORSES RUNNING.

Analysis of Record Performances on American Turf.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.
Distance, miles.	Time T, seconds.	Distance L, meters.	Speed V, $\frac{\text{meters}}{\text{seconds}}$.	log T.	log L.	log V.	log T' computed.	T' computed, seconds.	$T'-T$ discrepancy, seconds.	$100\left(\frac{T'-T}{T}\right)$ per cent discrepancy.	Mean per cent.
0.25	21.25	402.3	18.93	1.3274	2.6046	1.2772	1.3028	20.08	−1.17	5.5	
0.375	34.0	603.5	17.75	.5315	.7807	.2492	.5009	31.69	−2.31	6.8	
0.4375	40.5	704.1	17.38	.6075	.8476	.2401	.5761	37.68	−2.82	7.0	
0.5	46.0	804.7	17.49	.6628	.9056	.2428	.6414	43.79	−2.21	4.8	
0.5625	51.6	905.2	17.54	.7126	.9567	.2441	.6990	50.01	−1.59	3.1	
0.625	56.75	1005.8	17.72	.7540	3.0025	.2485	.7504	56.29	−0.46	0.8	
0.6875	62.5	1106.4	17.71	.7959	.0439	.2480	.7970	62.66	0.16	0.3	
0.75	68.0	1207.0	17.75	.8325	.0817	.2492	.8395	69.10	1.10	1.6	
0.8125	78.4	1307.6	16.68	.8943	.1165	.2222	.8787	75.63	−2.77	3.5	
0.875	83.5	1408.1	16.86	.9217	.1486	.2269	.9148	82.19	−1.31	1.6	
0.9375	92.2	1508.9	16.37	.9647	.1787	.2140	.9486	88.84	−3.36	3.6	
1.0000	95.5	1609.3	16.85	.9800	.2066	.2266	.9800	95.50	0	0	

1.0114	100.0	1627.6	16.28	2.0000	.2115	.2115	.9855	96.72	—3.28	3.3	1.9
1.028	101.2	1655.0	16.35	.0052	.2188	.2136	.9937	98.56	—2.64	2.6	
1.040	102.6	1673.3	16.31	.0111	.2236	.2125	.9990	99.98	—2.62	2.6	
1.0625	104.6	1709.9	16.35	.0195	.2330	.2135	2.0097	102.2	—2.4	2.3	
1.125	111.0	1810.5	16.31	.0453	.2578	.2125	.0376	109.0	—2.0	1.8	
1.1875	117.4	1911.1	16.28	.0697	.2813	.2116	.0741	118.6	1.2	1.0	
1.25	122.8	2011.7	16.38	.0892	.3036	.2144	.0891	122.8	0	0	
1.3125	133.0	2112.3	15.88	.1239	.3248	.2009	.1131	129.7	—3.3	2.5	
1.375	137.6	2212.9	16.08	.1386	.3450	.2064	.1357	136.7	—0.9	0.7	
1.5	150.25	2414.0	16.07	.1768	.3827	.2059	.1781	150.7	0.45	0.3	
1.625	165.2	2615.2	15.83	.2180	.4175	.1995	.2273	168.8	3.6	2.2	
1.75	177.0	2816.3	15.91	.2480	.4497	.2017	.2535	179.3	2.3	1.3	
1.875	199.0	3017.5	15.16	.2989	.4796	.1807	.2871	193.7	—5.3	2.6	
2.000	206.5	3218.6	15.59	.3149	.5077	.1928	.3188	208.4	1.9	0.9	
2.125	222.0	3419.8	15.40	.3464	.5340	.1876	.3483	223.0	1.0	0.5	
2.25	229.2	3621.0	15.80	.3602	.5588	.1986	.3762	237.8	8.6	3.8	
2.5	264.5	4023.3	15.21	.4224	.6046	.1822	.4278	267.8	—3.3	1.2	
2.625	298.5	4224.5	14.15	.4749	.6258	.1509	.4516	282.9	—15.6	5.2	
2.75	298.75	4425.7	14.82	.4753	.6460	.1707	.4743	298.1	—0.65	0.2	
3.00	324.0	4828.0	14.90	.5105	.6838	.1733	.5169	328.8	4.8	1.5	
4.00	431.0	6437.0	14.93	.6345	.8087	.1742	.6574	454.4	23.4	5.4	

FIGURE 2. *World's Trotting Records Plotted on Logarithm Paper.*

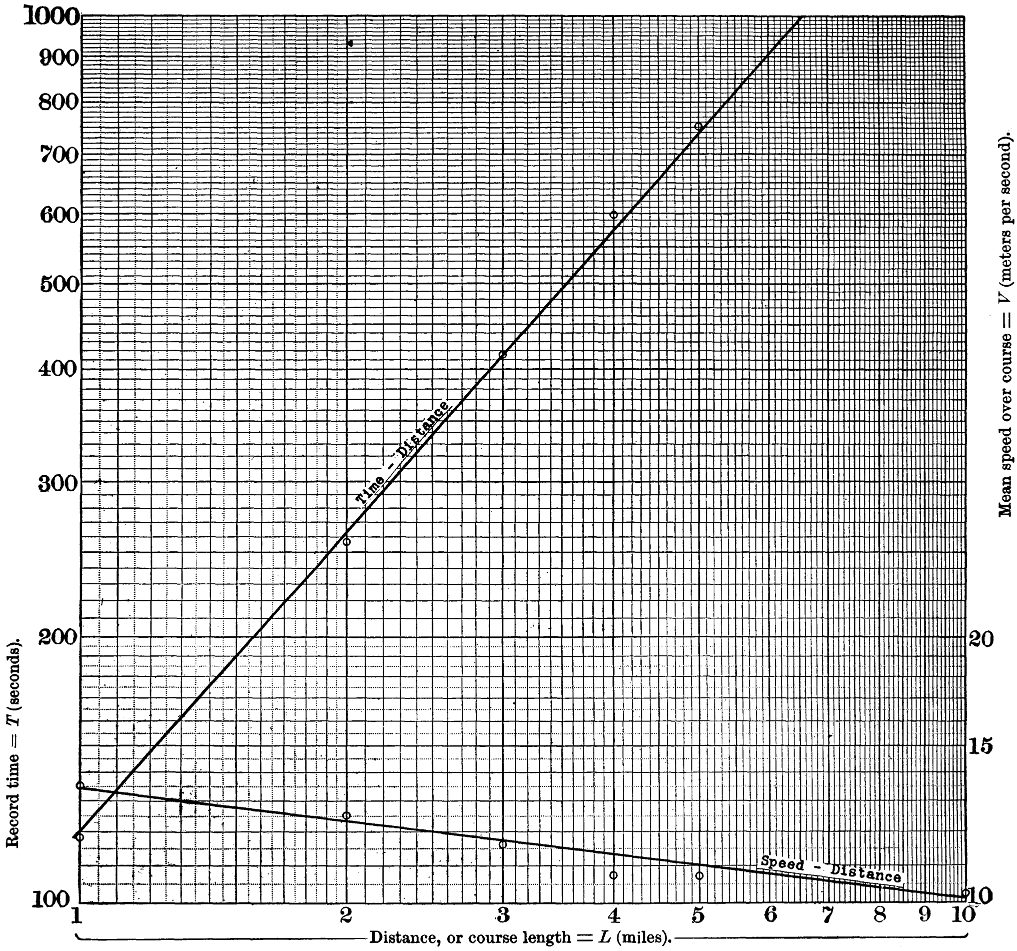


FIGURE 3. *World's Trotting Records to 100 Miles.*

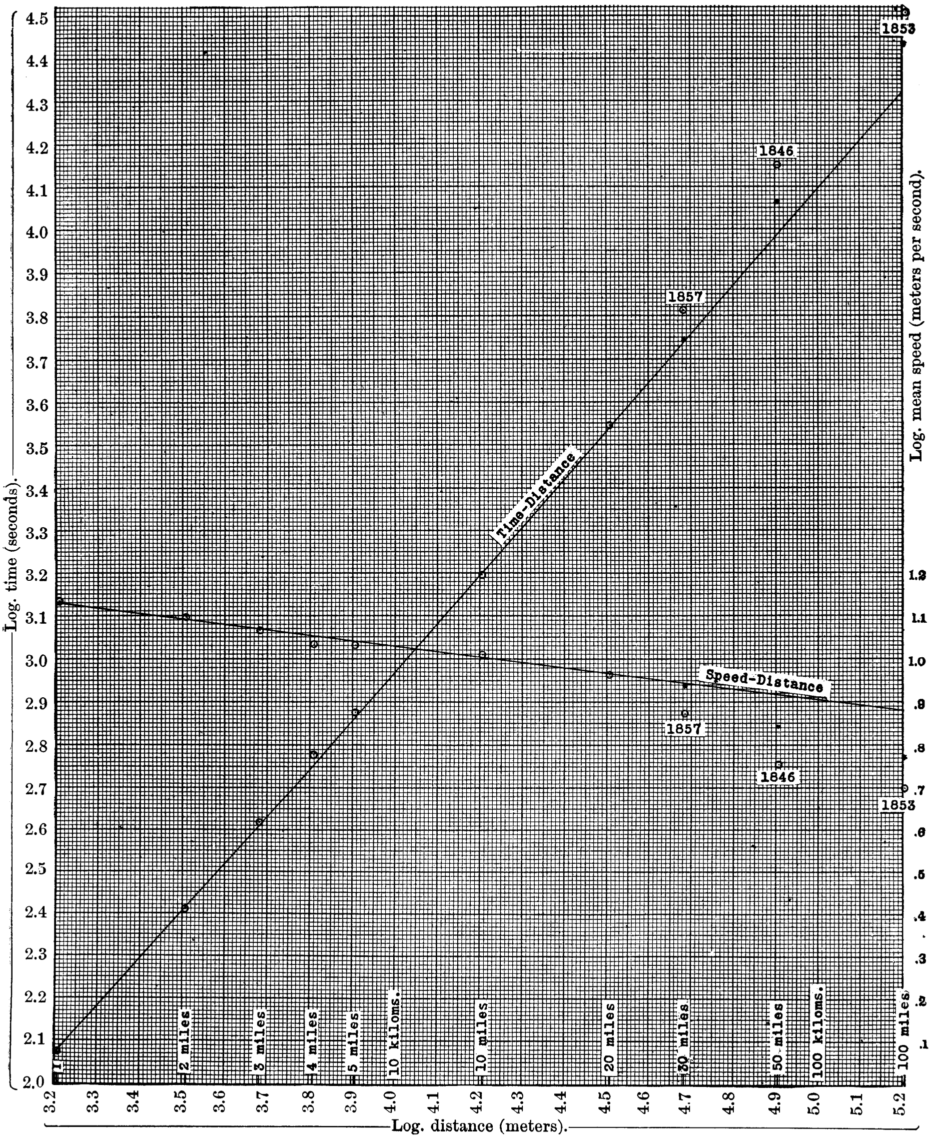


Figure 4. *World's Trotting Records to 100 Miles.*

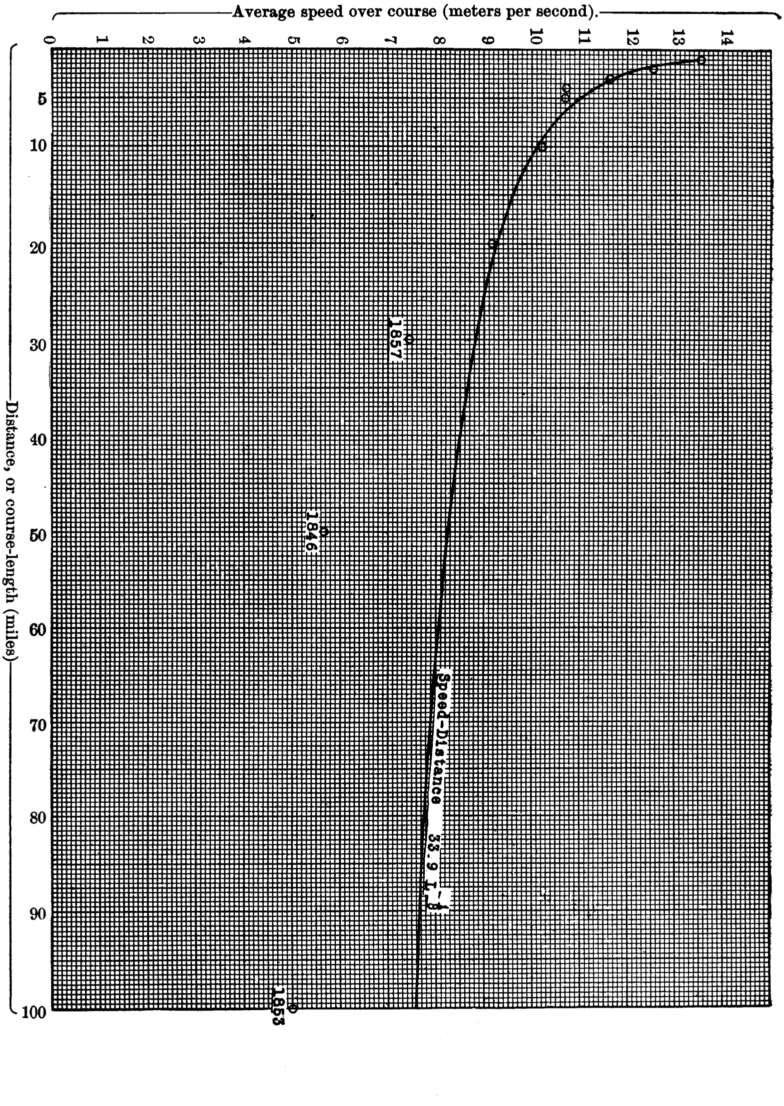


FIGURE 5. *Running Horses, Speeds and Record Times.*

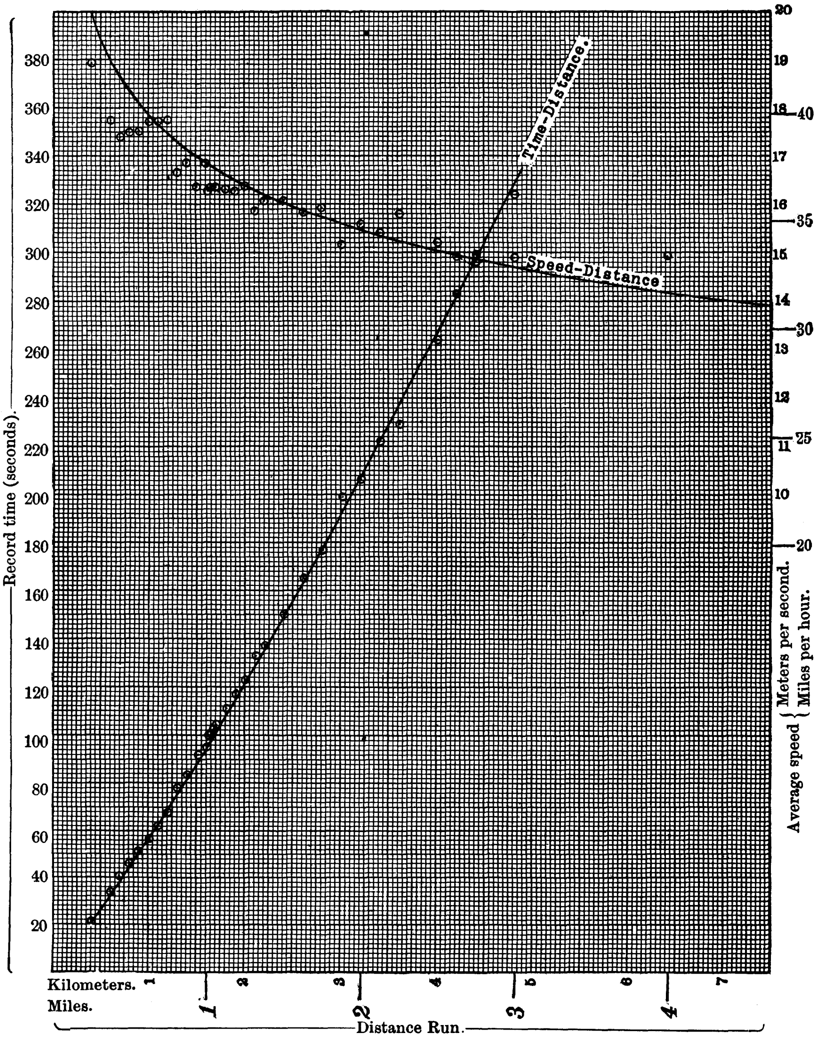
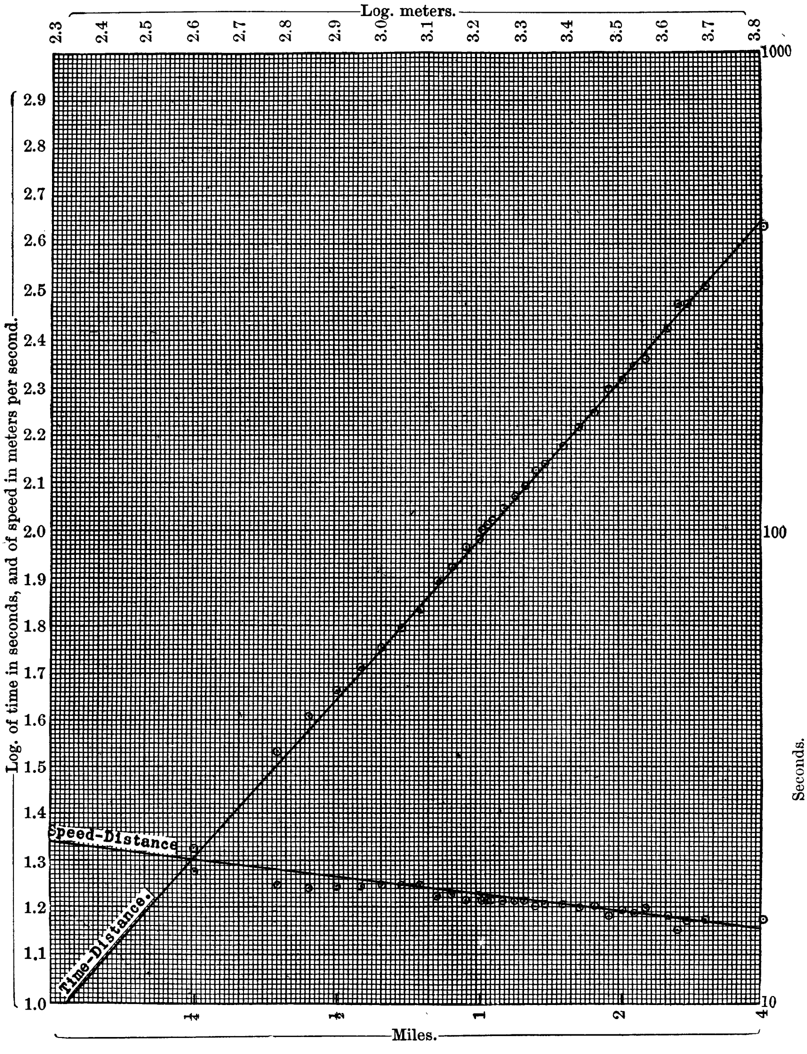


Figure 6. *Speeds and Record Times of Running Horses (Logarithmic Co-ordinates).*



Referring to Figure 6, it is to be observed that the record speeds are low, or the times high, with respect to the straight lines up to 1000 meters (0.62 mile). It is possible that this discrepancy on the short courses may be due to the inertia of the horses; that is to say, the horses may be supposed to lose time, or to miss attainment of speed, over the short runs below a kilometer, owing to the effort required to start their bodies into motion from rest. This time lost in acceleration is ignored in the straight-line logarithmic law, which assumes that the animal starts with full speed. For courses of over 1 kilometer, the discrepancy disappears.

Moreover, horse-races are stated to be rarely run over distances less than 5 furlongs (1006 meters); so that the question of the discrepancy over short courses is of but little practical importance.

The ascending straight line in Figure 6, drawn to meet the points as fairly as may be, is carried through the 1-mile record, and makes an angle with the axis of distance of 48° 22′, whose tangent is $\frac{9}{8}$. The falling line is also carried through the 1-mile record and makes an angle with the same axis of — 7° 7′ 30″, whose tangent is — ^{1}.

The straight lines of Figure 5 correspond to, or determine, the following equations:

$$\log T = \tfrac{9}{8} \log L - 1.6274 \tag{4}$$

and

$$\log V = 1.6274 - \tfrac{1}{8} \log L. \tag{5}$$

These are also respectively equivalent to:

$$T = \frac{L^{\frac{9}{8}}}{42.4} \quad \text{seconds} \tag{6}$$

and

$$V = \frac{42.4}{L^{\frac{1}{8}}} \quad \text{meters per second.} \tag{7}$$

By comparing equations (7) and (3) it appears from them that the record speeds of trotting horses is less than the record speeds of running horses over a given distance in the ratio of 33.9 to 42.4, or by 20 per cent of the latter.

On courses longer than 1 kilometer, the straight lines of Figure 6 fit the observations of Table II with satisfactory precision, some observations lying on one side and others on the other. The greatest discrepancies are at 2¼ miles and at 4 miles. Figure 5 shows in its upper line that both of these performances were remarkable. The speed over the 2¼ mile course appears from the data to have been greater than the speed in the 2-mile event. Again, the speed in the 4-mile race was actually higher than the speed in the 3-mile race.

Column IX of Table II gives the time that should correspond to each event, according to formulas (4) or (6). Column X gives the deviation from the record time. Column XI expresses the deviations in percentage of the record time. The mean percentage deviation between observed and computed record times between the limits of 1 kilometer (5 furlongs) and 6.44 kilometers (4 miles) is shown in column XII to be 1.9 per cent. Consequently, we may expect by following the ascending straight line of Figure 6 to predict the record time of any race between these limits to within 2 per cent on the average.

The mean percentage deviation between observed and computed record times for the entire series of events, i. e. between $\frac{1}{4}$ mile and 4 miles (0.4 to 6.44 kilometers), is 2.4 per cent.

Horses Pacing.

The record data for pacing horse-races in harness on American turf appear in columns I, II, and III of Table III. They are taken from page 260 of "The World Almanac" for 1906. The entries in the succeeding columns, IV to VIII, are then found in the manner previously described. In Figure 7 the logarithm of the time, or log T in column VII, is plotted against the logarithm of the distance, or log L in column VI. The logarithm of the mean speed in each event, or log V in column VIII, is also plotted. Taking the ascending time-distance line, it is a straight line ruled through the 2-mile record point, so as fairly to conform with the other record points. It makes an angle with the distance axis of 48° 22′ or $\tan^{-1}\frac{9}{8}$. Referring to the descending speed-distance line, it is a straight line drawn through the 2-mile record point, so as fairly to conform with the other record points. It makes an angle with the distance axis of $-7° \; 7' \; 30''$ or $\tan^{-1} - \frac{1}{8}$. The two straight lines include between them an angle of approximately 55° 30′ 30″.

The straight lines of Figure 7 correspond respectively to the following equations:

$$\log T = \tfrac{9}{8} \log L - 1.5363 \qquad (8)$$

$$\log V = 1.5363 - \tfrac{1}{8} \log L. \qquad (9)$$

These in turn correspond respectively to the following:

$$T = \frac{L^{\frac{9}{8}}}{34.38} \quad \text{seconds.} \qquad (10)$$

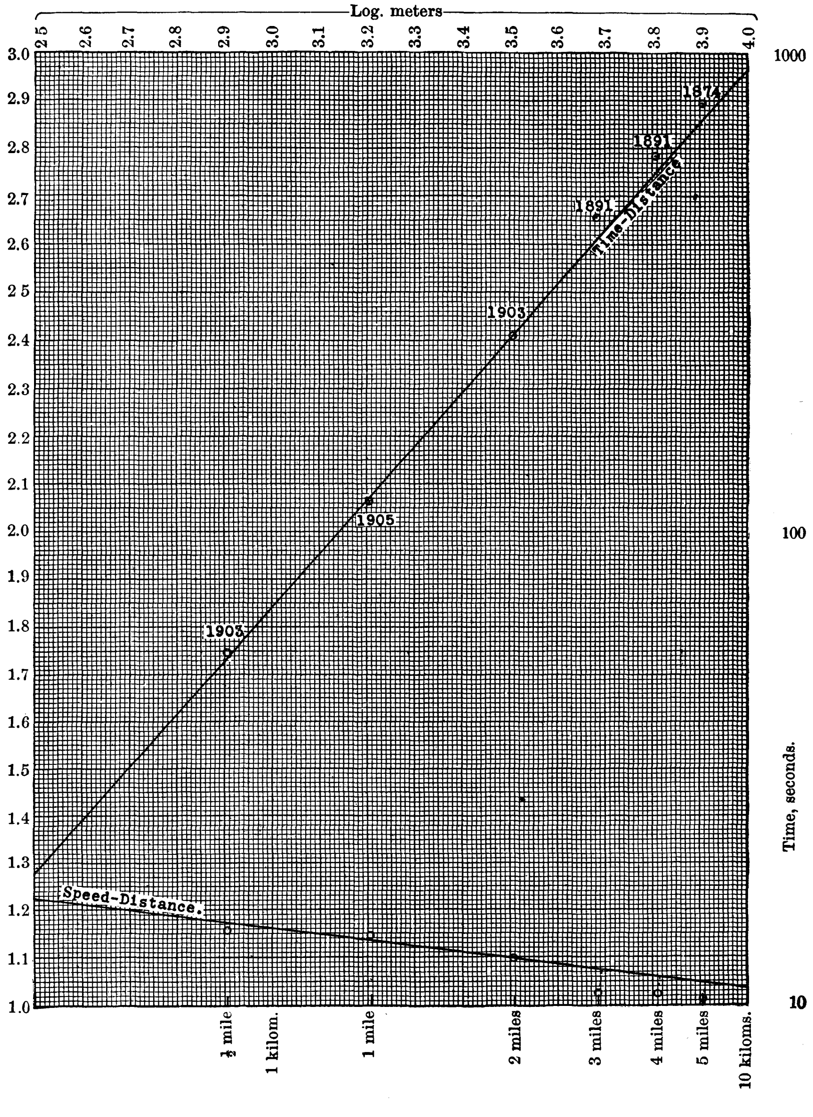
$$V = \frac{34.38}{L^{\frac{1}{8}}} \quad \text{meters per second.} \qquad (11)$$

TABLE III.

Analysis of Pacing Horse Records in Harness on American Turf.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.	
Date.	Distance, miles.	Time T, seconds	Distance L, meters.	Speed V, $\frac{\text{meters}}{\text{seconds}}$.	log L.	log T.	log V.	log T' computed.	T' computed, seconds	$T'-T$, deviation.	$100\left(\frac{T'-T}{T}\right)$ per cent deviation.	Means.
1903	½	56.0	804.7	14.37	2.9056	1.7482	1.1574	1.7325	54.0	−2.0	3.5	
1905	1	115.25	1609.3	13.97	3.2066	2.0616	1.1450	2.0711	117.8	2.55	2.2	1.9
1903	2	257.0	3218.7	12.53	3.5077	2.4099	1.0978	2.4099	257.0	0.0	0.0	
1891	3	453.25	4828.0	10.65	3.6838	2.6563	1.0275	2.6080	405.5	−47.75	10.5	5.4
1891	4	610.0	6437.4	10.55	3.8087	2.7853	1.0234	2.7485	560.4	−49.6	8.1	
1874	5	783.5	8046.7	10.27	3.9056	2.8940	1.0116	2.8575	720.3	−63.2	8.1	

FIGURE 7. *Pacing Records Plotted to Logarithmic Co-ordinates.*



An inspection of column X, Table III, containing the computed record times according to formulas (8) and (10), shows that the ½-mile record was 2 seconds longer than the computed time, the mile 2½ seconds shorter, and the 3-mile, 4-mile, and 5-mile events considerably longer. These deviations appear in column XI, and are given in percentages of the respective actual records in column XII. The ½-mile deviation of 3½ per cent may be explained by inertia on short courses. The deviations of 10½, 8, and 8 per cent on the long distances are not explainable in such a manner. The 3-mile and 4-mile records date from 1891, and the 5-mile record from 1874. Reference to the records of 1892, as given on page 273 of "The World Almanac" for 1894, shows that at that time the pacing records were low by comparison with those at the present date. Thus the mile record (1.6 kilometers) in pacing was 124 seconds, and the 2-mile record (3.2 kilometers) was 287.75 seconds, which are in deviation from the modern records of those events, according to Table III, by 7.6 and 11.9 per cent respectively. Consequently, it seems fair to say that these records of 3, 4, and 5 miles, which are 8 to 10.5 per cent off the straight lines in Figure 7, would have been close to similar lines drawn for records in 1894. In other words, the 1-mile and 2-mile records have steadily improved since 1891, whereas the 3-mile, 4-mile, and 5-mile records remain as they were at that date.

If we eliminate the 3-mile, 4-mile, and 5-mile records from consideration, the average discrepancy between the computed and observed record times is seen in column XII to be 1.9 per cent. If, however, we have to consider all of the events, the average discrepancy is 5.4 per cent.

Summing up the analysis of horse-racing as presented in Tables I, II, and III with their accompanying curve-sheets, it is submitted that the logarithmic straight lines meet the observations, within reasonable limits of range, to 2 per cent of average discrepancy. Moreover, these corresponding straight logarithmic lines are parallel to each other for running, pacing, or trotting. This means that there is a similar law of fatigue in each of these styles of progression.

Men Running.

The first two columns of Table IV give the world's records of running races. The first four events, up to 45 yards inclusive, are taken from the Amateur Athletic Union Records, as given on page 262 of "The World Almanac" for 1906. The remaining events, up to 623 miles inclusive, are taken from page 242 of "The World Almanac" for 1904. For each event the best record has been selected, whether the holder

TABLE IV. — Men Running.

Analysis of Best Professional and Amateur World's Records.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.
Distance, yards.	Time T, seconds.	Distance L, meters.	Speed V, $\frac{\text{meters}}{\text{seconds}}$.	log L.	log T.	log V.	T' computed.	$T'-T$ deviation, seconds.	$100\left(\frac{T'-T}{T}\right)$ deviation per cent.
20	A 2.8	19.5	5.188	1.2622	0.4472	0.7150	1.55	–1.25	–44.5
35	A 4.0	32.0	8.000	.5052	.6021	.9031	2.90	–1.1	–27.5
40	A 4.4	36.6	8.312	.5632	.6435	.9197	3.37	–1.03	–23.5
45	A 5.2	41.2	7.914	.6144	.7160	.8984	3.85	–1.35	–26.0
50	5.25	45.7	8.706	.6600	.7202	.9398	4.33	–0.92	–17.5
60	A 6.4	54.9	8.572	.7393	.8062	.9331	5.32	–1.08	–16.9
70	A 7.2	64.0	8.890	.8062	.8573	.9489	6.33	–0.87	–12.1
75	7.25	68.6	9.458	.8362	.8604	.9758	6.84	–0.41	–5.7
80	A 8.0	73.2	9.145	.8643	.9031	.9612	7.36	–0.64	–8.0
100	9.6	91.4	9.524	.9611	.9823	.9788	9.5	–0.1	–1.0
110	11.0	100.6	9.144	2.0025	1.0414	.9611	10.5	–0.5	–4.5
120	A 11.4	109.7	9.625	.0403	.0569	.9834	11.6	0.2	1.8
125	A 12.4	114.3	9.207	.0580	.0934	.9646	12.2	–0.2	–1.6
130	12.125	118.9	9.804	.0751	.0837	.9914	12.7	0.6	4.8
131.5	12.4	120.2	9.696	.0800	.0934	.9866	12.9	0.5	4.0
135	13.2	123.4	9.352	.0915	.1206	.9709	13.3	0.1	0.8
140	13.5	128.0	9.482	.1072	.1303	.9769	13.8	0.3	2.2
150	14.5	137.2	9.458	.1372	.1614	.9758	14.9	0.4	2.8
180	A 18.0	164.6	9.144	.2164	.2553	.9611	18.3	0.3	1.7
200	19.5	182.9	9.380	.2622	.2900	.9722	20.6	1.1	5.7
220	A 21.2	201.2	9.490	.3036	.3263	.9773	23.0	1.8	8.5
250	A 24.6	228.6	9.294	.3591	.3909	.9682	26.5	1.9	7.7
300	30.0	274.3	9.144	.4382	.4771	.9611	32.5	2.5	8.3
350	A 36.4	320.0	8.792	.5052	.5611	.9441	38.7	2.3	6.3
400	A 42.2	365.8	8.668	.5632	.6253	.9379	45.0	2.8	6.6
440	A 47.0	402.3	8.561	.6046	.6721	.9325	50.1	3.1	6.6
500	A 57.8	457.2	7.910	.6601	.7619	.8982	57.8	0	0
600	A 71.0	548.6	7.727	.7393	.8513	.8880	71.0	0	0
660	A 82.0	603.5	7.361	.7807	.9138	.8669	79.0	–3.0	–3.7
700	89.0	640.1	7.203	.8069	.9494	.8575	86.5	–2.5	–2.8
800	A 104.4	731.4	7.007	.8642	2.0187	.8455	98.0	–6.4	–6.1
880	113.5	804.7	7.089	.9056	.0550	.8506	109.2	–4.3	–3.8
1000	A 133.0	914.4	6.874	.9611	.1239	.8372	126.0	–7.0	–5.3
1320	A 182.8	1207.0	6.602	3.0817	.2620	.8197	172.2	–10.6	–5.8
miles									
1	252.75	1609.3	6.367	.2066	.4027	.8039	238.1	–14.6	–5.8
1.25	330.0	2011.6	6.095	.3035	.5185	.7850	299.0	–31.0	–9.4
1.5	403.5	2414.0	5.997	.3837	.6058	.7779	376.7	–26.8	–6.6
1.75	A 488.2	2816.0	5.768	.4496	.6886	.7610	446.8	–41.4	–8.5
2	551.5	3218.7	5.837	.5077	.7415	.7662	519.4	–32.1	–5.8
2.5	A 726.0	4023.0	5.542	.6046	.8609	.7437	667.6	–58.4	–8.0
3	859.5	4828.0	5.618	.6838	.9342	.7496	819.6	–39.9	–4.6
3.5	1004.2	5633.0	5.607	.7507	3.0020	.7487	974.6	–29.6	–3.0
4	1165.6	6437.0	5.525	.8088	.0665	.7423	1133.0	–32.6	–2.8
4.5	A 1345.0	7242.0	5.385	.8599	.1287	.7312	1293.0	–52.0	–3.9
5	1480.0	8047.0	5.436	.9056	.1703	.7353	1456.0	–24.0	–1.6
5.5	A 1662.6	8851.0	5.150	.9470	.2208	.7262	1621.0	–41.6	–2.5
6	1790.0	9656.0	5.394	.9848	.2529	.7319	1787.0	–3.0	–0.2
6.5	A 1976.4	10,462.0	5.293	4.0196	.2959	.7237	1956.0	–20.4	–1.0
7	2085.0	11,265.0	5.404	.0518	.3191	.7327	2126.0	41.0	2.0

TABLE IV. — *Continued.*

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.
Distance, miles.	Time T, seconds.	Distance L, meters.	Speed V, $\frac{\text{meters}}{\text{seconds}}$.	log L.	log T.	log V.	T' computed.	$T'-T$ deviation, seconds.	$100\left(\frac{T'-T}{T}\right)$ deviation per cent.
7.5	A 2298.0	12,070	5.252	4.0817	3.3614	0.7203	2297	–1	0
8	2420.0	12,875	5.320	.1097	.3838	.7259	2470	50	2.1
8.5	A 2613.0	13,680	5.236	.1361	.4171	.7190	2645	32	1.2
9	2721.0	14,484	5.322	.1608	.4347	.7261	2820	99	3.7
9.5	A 2931.0	15,289	5.217	.1844	.4670	.7174	2998	67	2.3
10	3066.6	16,094	5.248	.2066	.4866	.7200	3175	108.4	3.5
10.5	3229.0	16,900	5.214	.2279	.5097	.7182	3355	126	3.9
11	3388.0	17,703	5.225	.2480	.5299	.7181	3534	146	4.3
11.5	3543.0	18,508	5.224	.2674	.5494	.7180	3716	173	4.9
12	3722.5	19,312	5.187	.2858	.5709	.7149	3898	175.5	4.7
13	4231.0	20,922	4.946	.3206	.6264	.6942	4266	35	0.8
14	4572.0	22,531	4.928	.3528	.6601	.6927	4637	65	1.4
15	A 4804.6	24,140	5.023	.3827	.6817	.7010	5010	205.4	4.3
16	5286.0	25,750	4.872	.4108	.7231	.6877	5389	103	1.9
17	5655.0	27,360	4.838	.4371	.7524	.6847	5768	113	2.0
18	A 6010.0	28,970	4.819	.4619	.7789	.6830	6147	137	2.3
19	A 6360.0	30,580	4.807	.4854	.8035	.6819	6537	177	2.8
20	A 6714.0	32,190	4.794	.5077	.8270	.6807	6926	212	3.1
21	A 7570.0	33,790	4.464	.5288	.8791	.6497	7315	–255	–3.4
22	A 7968.0	35,410	4.444	.5491	.9013	.6478	7709	–259	–3.2
23	A 8390 0	37,010	4.411	.5683	.9238	.6445	7918	–472	–5.6
24	A 8825.0	38,620	4.377	.5869	.9457	.6412	8504	–321	–3.6
25	A 9224.0	40,240	4.363	.6047	.9649	.6398	8904	–320	–3.5
30	11,709.0	48,280	4.124	.6838	4.0685	.6153	10,930	–779	–6.6
40	16,647.0	64,370	3.868	.8088	.2214	.5874	15,110	–1537	–8.6
50	21,304.5	80,470	3.777	.9056	.3285	.5771	19,410	–1895	–8.9
60	27,033.0	96,560	3.572	.9848	.4319	.5529	23,830	–3203	–11.9
70	32,595.0	112,650	3.456	5.0518	.5132	.5386	28,350	–4245	–13.0
80	38,030.0	128,750	3.386	.1098	.5801	.5297	32,950	–5080	–13.4
90	43,215.0	144,850	3.352	.1609	.6356	.5253	37,610	–5605	–13.0
100	48,390.0	160,933	3.325	.2066	.6848	.5218	43,320	–5070	–10 5
110	55,245.0	177,030	3.204	.2480	.7423	.5057	47,130	–8115	–14.7
120	60,490.0	193,120	3.192	.2858	.7817	.5041	51,980	–8510	–14.1
130	68,685.0	209,220	3.046	.3206	.8369	.4837	56,890	–11,795	–17.2
140	75,030.0	225,310	3.003	.3528	.8752	.4776	61,830	–13,200	–17.6
150	80,905.0	241,400	2.984	.3828	.9080	.4748	68,380	–12,525	–16.7
200	126,568.0	321,900	2.543	.5077	5.1024	.4053	92,370	–34,200	–27.0
300	209,826.0	482,800	2.302	.6838	.3218	.3620	145,700	–64,126	–25.8
383	288,625.0	616,400	2.135	.7899	.4606	.3293	192,300	–96,325	–33.4
450	343,578.0	724,210	2.108	.8599	.5361	.3238	230,000	–113,578	–33.0
500	393,509.0	804,700	2.005	.9056	.5949	.3107	258,900	–134,600	–34.1
560	451,485.0	901,200	1.996	.9548	.6547	.3001	294,100	–157,385	–34.8
623	510,030.0	1,003,000	1.966	6.0013	.7078	.2935	331,700	–179,300	–34.5
. .	7.0	60	8.57	1.7782	0.8451	.9331			
. .	10.8	100	9.25	2.0000	1.0334	.9666			
. .	21.6	200	9.25	2.3010	1.3345	.9665			
. .	49.2	400	8.13	2.6021	1.6920	.9101			
. .	116.0	800	6.90	2.9031	2.0645	.8386			
. .	245.8	1500	6.10	3.1761	2.3906	.7855			
5	1571.6	8047	5.12	3.9056	3.1962	.7094			
. .	10,288.0	40,000	3.90	4.6021	4.0123	.5898			

was amateur or professional. Amateur records are indicated in column II by the letter A. The last eight events are taken from records of the Olympian Games, as published in the Boston "Transcript" for May 12, 1906. These international contests have been held in 1896, 1900, 1904, and 1906. The best of these four records has been taken for each event.

In Figure 8, log T (see column VI, Table IV) is plotted as ordinates against log L (column V) as abscissas. The circles mark the world's records and the crosses the Olympian records. In order to keep the chart within reasonably small dimensions, retaining a fairly extended scale, the observations are made to cross the chart twice, by employing dual scales of ordinates and abscissas. The straight line is drawn through the 500-yard record to meet the remaining observations. It runs off the sheet the first time at log $T = 2.71$ and log $L = 3.5$. It then recommences at the left hand of the upper line and finally leaves the sheet at log $T = 4.9$ and log $L = 5.45$. These parts of what would be a single straight line on a larger sheet make an angle of 48°22′ or $\tan^{-1}\frac{9}{8}$ with the axis of abscissas. The record points conform closely to this line, swinging slightly from one side of it to the other. They lie above it as far as log $L = 2.0$. They lie beneath from log $L = 2.0$ to log $L = 2.66$, and again from log $L = 4.0$ to 4.5. In the remaining parts they lie above. In other words, the straight line hits the record path at log $L = 2.1$, 2.7, 3.75, and 4.5.

Table IV gives in column VIII the values of the record times corresponding to the straight line in Figure 8. Column IX gives the deviation from the world's record for each event and column X the percentage deviation. The percentage deviation commences at —44.5 for the shortest run (19.5 meters). It dwindles to 0 at about 110 meters. It reaches a maximum of 8.5 near 200 meters, returns to 0 at 500 meters, swings over to —8.5 at 2800 meters, returns to 0 near 10 kilometers, swings to 4.9 at 18 kilometers, crosses the zero point near 30 kilometers, and then steadily increases numerically and in the negative direction until it is —34.8 at 900 kilometers.

The large discrepancies below 100 yards may be attributed to inertia. This is indicated in Figure 9, where the speeds of the events are plotted up to 201 meters. It is evident from the dotted line following the records that runners attain their maximum apparent speed in the neighborhood of 110 meters (120 yards). At shorter distances the retarding effect of inertia prevents a higher average speed in the run from being developed. At longer distances, the starting retardation is reduced in effect, but fatigue acts in its place. The simple logarithmic law, represented by the heavy line, takes no account of inertia and assumes a maximum speed on an indefinitely short course. It is probably not worth while going to the complication of correcting the formula by a term or terms introducing a retardation at the start, since races below 100 yards are rare, and the deviation between the actual and computed speeds disappears beyond 100 yards.

At the other end of the line in Figure 8 the deviations become large after $\log L = 5$ or beyond 100 kilometers. Beyond 150 miles the deviations exceed 20 per cent. It is possible that the discrepancy may here be accounted for by reason of the fact that somewhere in this neighborhood the runner stops at intervals to take food, or rest, and there is no longer a continuous performance enacted. According to the logarithmic straight line, the speeds on courses between 450 and 623 miles (724 and 1003 kilometers) are only two-thirds of what should be expected.

Summing up the entries in column X, without regard to sign, it is found that if all the events are included from 19.5 meters to 1003 kilometers, the average deviation is 8.9 per cent. If, however, the summation be limited to the range from 100 meters to 100 kilometers, the average deviation is 4.3 per cent. This appears to be a satisfactory result, considering that the range of distance is 1000 to 1, and the range of record times 2500 to 1.

From a practical standpoint it may be inferred from an inspection of either Table IV or Figure 8, that it should be easier for trained athletes to beat the world's records between 600 meters and 9 kilometers, or between 30 and 1000 kilometers, than to beat the records between 100 meters and 600 meters, or between 10 kilometers and 30 kilometers. Expressed in another way, we should expect the degree of physical exhaustion in record runs over the short courses up to 600 meters to be more severe than on the courses from that distance up to 9 kilometers. The existing one-mile or three-mile record does not seem to be so severe as the records from 100 yards to 500 yards. Whatever mathematical conclusions may be drawn from the data, the belief seems unavoidable that a study of Figure 8 will be useful to athletes training for running with a view to breaking records.

The crosses in Figure 8, representing Olympian records, while useful as a check, do not serve the results in establishing the straight lines, because these Olympian records are inferior to the corresponding world's records, excepting that for the 100-meter race.

The line of Figure 8 corresponds to the following formulas :

$$\log T = \tfrac{9}{8} \log L - 1.2307 \qquad (12)$$

FIGURE 8. *World's Running Records.*

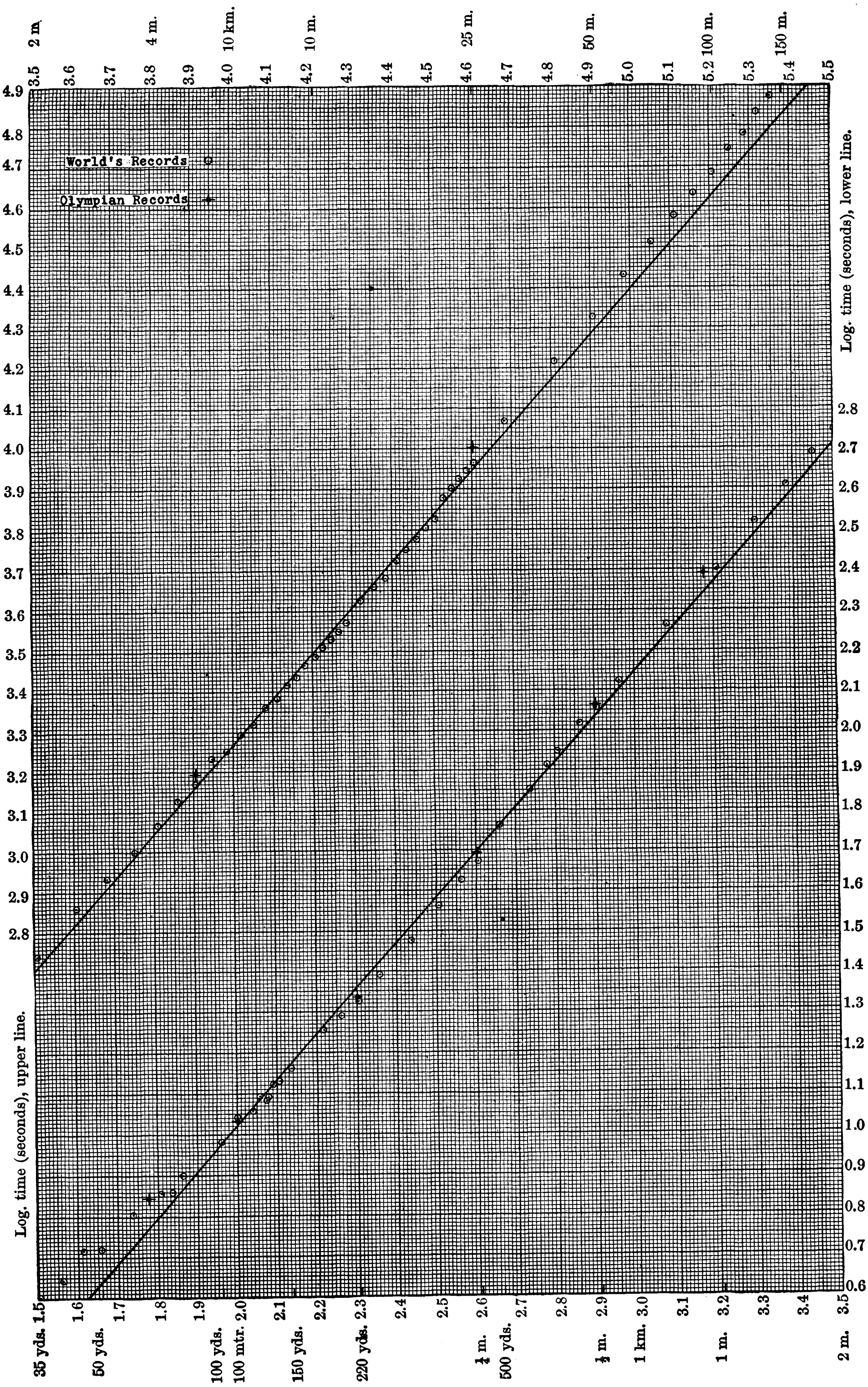
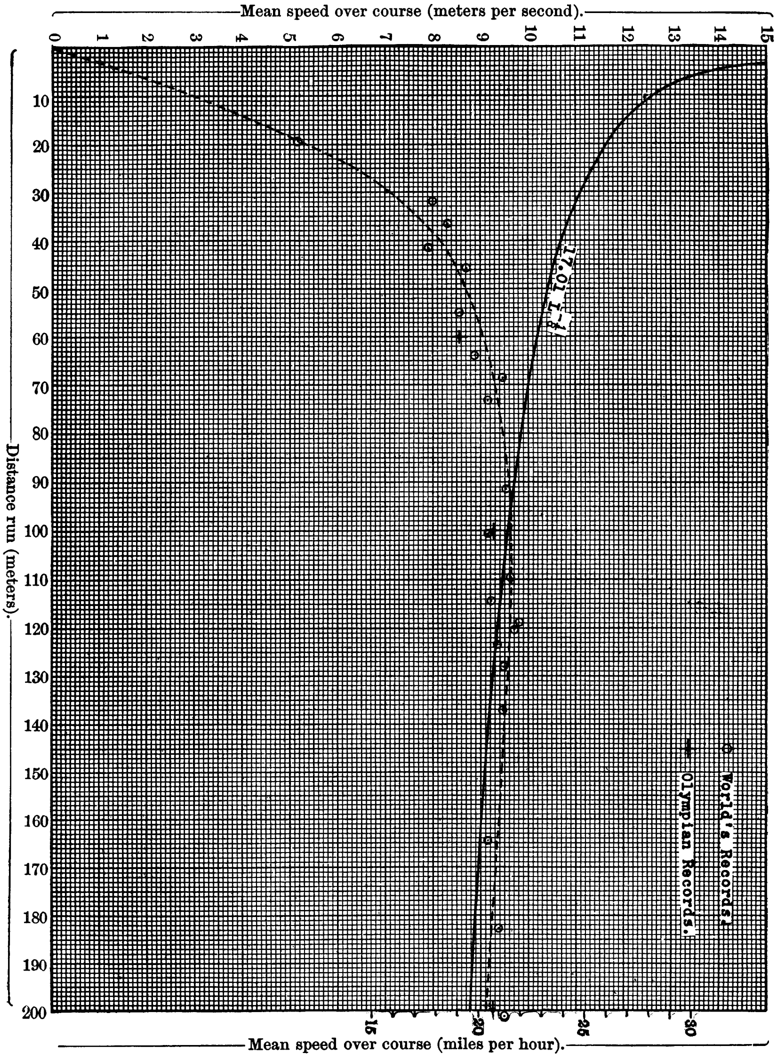


FIGURE 9. *Mean Running Speeds over Short Courses.*



NOTE. — Figure 8, on the opposite page, expresses the relation between time and distance to logarithmic co-ordinates. The scale of abscissas at the top of the figure gives the logarithm of distance in meters for the upper line; that at the bottom, for the lower line.

or
$$T = \frac{L^{\frac{9}{8}}}{17.01} \text{ seconds.} \tag{13}$$

$$\log V = 1.2307 - \tfrac{1}{8} \log L \tag{14}$$

or
$$V = \frac{17.01}{L^{\frac{1}{8}}} \text{ meters per second.} \tag{15}$$

MEN WALKING.

The data for walking races are given in columns I and III of Table V. They are taken from page 244 of "The World Almanac" for 1904, and are world's records, amateur and professional. The best records in walking, unlike other athletic sports, seem to have been made entirely by professional walkers.

Figure 10 indicates the points found by plotting log T from column V against log L from column II. In order to economize space, the series of points is carried twice across the sheet to two sets of scales. The straight line, seen in two segments, is drawn through the 4-mile record point, and is drawn to meet the other points fairly. It makes an angle of 48° 22′ or $\tan^{-1} \frac{9}{8}$, with the axis of distances. The entries in column VIII correspond to points on the straight line. The computed times T' of each event are thus obtained and set down in column IX. The last column gives the percentages of deviation between the actual and computed record. The observations commence above the line, or the record times for the 1-mile and 2-mile events are long, or the speeds low, by comparison with the 4-mile and the 50-mile records. This can hardly be accounted for by inertia, as in horse-racing; because at the low speed of walking, the retardation, due to starting from rest, must disappear in less than 100 meters.

The points fall below the line between 4 miles and 30 miles, representing faster speeds than the computed to the extent of nearly 6 per cent. Between 40 and 70 miles the agreement between the observed and computed times is close. Beyond 70 miles the speed falls below that computed by the logarithmic line. The deviations do not exceed 5 per cent until beyond 120 miles.

The mean percentage of deviation, without regard to sign, is 5.6 per cent over the entire series of events, and 3.4 per cent between the limits of 1 mile and 120 miles.

There seems to have been scarcely any improvement in walking records during the ten years preceding the date at which these have been selected (1904).

FIGURE 10. *World's Walking Records.*

Time-distance to logarithmic co-ordinates.

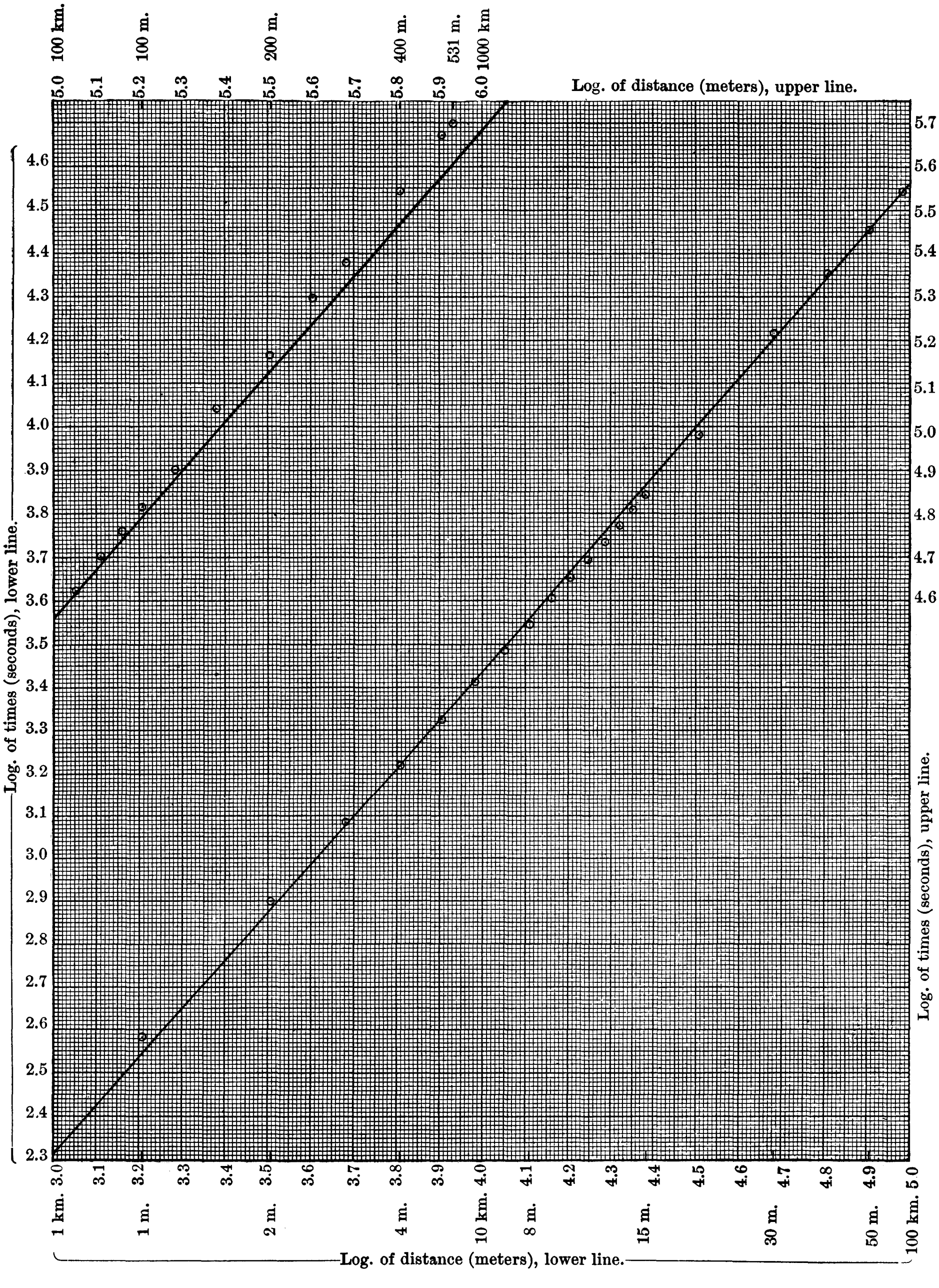


TABLE V.

MEN WALKING.

Analysis of World's Best Walking Records, Amateur or Professional.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.
Distance, miles.	Distance, L, meters.	Time, T, seconds.	log L.	log T.	log V.	V, $\frac{\text{meters}}{\text{seconds}}$.	log T', computed.	T', computed.	$T'-T$ deviation.	$100\left(\frac{T'-T}{T}\right)$ percentage deviation.
1	1,609.4	383.0	3.2066	2.5832	0.6234	4.201	2.5428	349.0	–34.0	–8.9
2	3,218.7	794.0	.5077	2.8998	.6079	.054	.8816	761.4	–32.6	–4.1
3	4,828.0	1,221.5	.6838	3.0869	.5969	3.953	3.0797	1,201.0	–20.5	–1.7
4	6,437.4	1,658.0	.8087	.2196	.5891	.882	.2202	1,660.0	2.0	0.0
5	8,046.7	2,110.0	.9056	.3243	.5813	.813	.3292	2,134.0	24.0	1.1
6	9,656.1	2,581.0	.9848	.4118	.5730	.741	.4183	2,620.0	39.0	1.2
7	11,265.0	3,064.0	4.0517	.4863	.5654	.676	.4936	3,116.0	52.0	1.7
8	12,875.0	3,517.0	.1097	.5462	.5635	.660	.5588	3,621.0	104.0	3.0
9	14,484.0	4,034.0	.1608	.6057	.5551	.590	.6163	4,133.0	99.0	2.5
10	16,094.0	4,485.0	.2066	.6518	.5548	.588	.6678	4,654.0	169.0	3.8
11	17,703.0	4,958.0	.2480	.6953	.5527	.570	.7144	5,181.0	223.0	4.5
12	19,312.0	5,434.0	.2858	.7351	.5507	.554	.7569	5,713.0	279.0	5.1

13	20,922.0	5,926.5	.3206	.7727	.5479	.531	.7961	6,253.0	326.5	5.5
14	22,531.0	6,431.5	.3528	.8083	.5445	.504	.8323	6,797.0	365.5	5.7
15	24,140.0	6,956.0	.3827	.8424	.5403	.470	.8659	7,343.0	387.0	5.6
20	32,187.0	9,597.0	.5077	.9821	.5256	.354	4.0066	10,150.0	553.0	5.8
30	48,280.0	16,494.0	.6838	4.2173	.4665	2.928	.2047	16,110.0	—384.0	—2.3
40	64,374.0	22,610.0	.8087	.3543	.4544	.847	.3452	22,140.0	—470.0	—2.1
50	80,467.0	28,456.0	.9056	.4542	.4514	.828	.4542	28,460.0	4.0	0.0
60	96,561.0	34,847.0	.9848	.5422	.4426	.771	.5433	34,940.0	93.0	0.3
70	112,650.0	41,915.0	5.0517	.6224	.4293	.687	.6186	41,550.0	—365.0	—0.9
80	128,750.0	50,513.0	.1097	.7034	.4063	.549	.6838	48,280.0	—2,233.0	—4.4
90	144,840.0	57,550.0	.1608	.7600	.4008	.517	.7413	55,120.0	—2,430.0	—4.2
100	160,940.0	65,330.0	.2066	.8151	.3915	.463	.7928	62,060.0	—3,270.0	—5.0
120	193,120.0	79,585.0	.2858	.9009	.3849	.426	.8819	76,190.0	—3,395.0	—4.3
150	241,400.0	110,188.0	.3827	5.0421	.3406	.191	.9909	97,930.0	—12,258.0	—11.1
200	321,870.0	146,790.0	.5077	.1667	.3410	.193	5.1316	135,400.0	—11,390.0	—8.3
250	402,340.0	198,744.0	.6046	.2983	.3063	.024	.2406	174,000.0	—24,744.0	—12.5
300	482,800.0	239,400.0	.6838	.3791	.3047	.017	.3297	213,500.0	—25,900.0	—10.8
400	643,740.0	348,663.0	.8087	.5424	.2663	1.846	.4702	295,300.0	—53,363.0	—15.3
500	804,670.0	470,025.0	.9056	.6721	.2335	.712	.5792	379,500.0	—90,525.0	—19.3
531	854,559.0	499,710.0	.9317	.6987	.2328	.709	.6086	406,100.0	—93,610.0	—17.8

The straight line of Figure 10 corresponds to the following equations:

$$\log T = \tfrac{9}{8} \log L - 1.0646 \qquad (16)$$

and

$$\log V = 1.0646 - \tfrac{1}{8} \log L \qquad (17)$$

or

$$T = \frac{L^{\frac{9}{8}}}{11.6} \quad \text{seconds} \qquad (18)$$

and

$$V = \frac{11.6}{L^{\frac{1}{8}}} \quad \text{meters per second.} \qquad (19)$$

MEN ROWING.

Table VI is compiled from the data contained in the article on "Rowing" appearing on page 208, Vol. X, of the "Universal Encyclopedia" published in 1900. The entries in columns I, II, III, and V of table VI are taken directly from that article. The remaining columns give the deductions therefrom, as in preceding cases. The logarithms of the times are plotted against the logarithms of the distances in Figure 11. The 8-oar line is drawn through the 4-mile record point. The 1-mile record is the only point seriously off the line, in the direction of low speed. The 1-mile speed is seen in column VI to be lower than the 4-mile speed. The 4-oar line is also drawn through the 4-mile record point. The 3-mile event is the only one seriously off this line, in the direction of high speed. The speed over this course was 5.15 meters per second, which is only about 7 per cent short of the record speed for the 1-mile event. The single-pair-of-sculls line is drawn through the 5-mile record point. The ¼-mile and the 1-mile events are the only ones seriously off this line. It is possible that the quarter-mile is affected by starting inertia. The mean deviation for the entire series is 6 per cent. If we discard the 1-mile 8-oar event and the ¼-mile singles event, the mean deviation of the remaining series is 4.3 per cent. This seems to be a good showing for the logarithmic straight line considering the extent to which both wind and tide are capable of influencing rowing speeds.

All three straight lines in Figure 11 are drawn to make an angle of 48° 22′ with the axis of distances.

The formulas deducible from the three straight lines of Figure 11 are presented in Table VII.

TABLE VI

MEN ROWING.

Analysis of Records of American Oarsmen.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.	XIII.
Type.	Date.	Distance, miles.	Distance L, meters.	Time T, seconds.	Speed V, $\frac{\text{meters}}{\text{seconds}}$.	log L.	log T.	log V.	log T' computed.	T' computed, seconds.	$T'-T$ deviation, seconds.	$100\left(\frac{T'-T}{T}\right)$ per cent deviation.
8-oars	1883	1	1609.3	304.75	5.281	3.2066	2.4839	0.7227	2.4054	254.3	−50.5	−16.5
	1889	1.5	2414	400	6.035	.3827	.6021	.7806	.6035	401.3	1.3	0.3
	1891	2	3218.7	583.5	5.516	.5077	.7660	.7417	.7442	554.9	−28.6	−4.9
	1891	3	4828	867.5	5.564	.6838	.9383	.7455	.9423	875.6	8.1	0.9
	1888	4	6437.4	1210	5.320	.8087	3.0828	.7259	3.0828	1210	0	0
4-oars	1883	1	1609.3	291	5.530	3.2066	2.4639	0.7427	2.4929	311.1	20.1	6.9
	1887	2	3218.7	747	4.309	.5077	.8733	.6344	.8317	678.8	−68.2	−9.1
	1875	3	4828	937.25	5.151	.6838	.9719	.7119	3.0298	1071	133.8	14.6
	1871	4	6437.4	1480	4.349	.8087	3.1703	.6384	.1703	1480	0	0
	1860	5	8046.7	1844.75	4.362	.9056	.2659	.6397	.2793	1902	57.3	3.1
	1871	6	9656.1	2360.6	4.090	.9848	.3730	.6118	.3684	2390	29.4	1.2
single-sculls	1891	¼	402.3	57	7.058	2.6046	1.7559	0.8487	1.8352	68.4	11.4	20.0
	1872	1	1609.3	301	5.347	3.2066	2.4786	.7280	2.5224	333	32.0	9.6
	1868	2	3218.7	680	4.734	.5077	.8325	.6752	.8612	726.4	46.4	6.8
	1890	3	4828	1171	4.123	.6838	3.0686	.6152	3.0593	1146	−25.0	−2.1
	1883	4	6437.4	1677.5	3.837	.8087	.2247	.5840	.200	1585	−92.5	−5.5
	1879	5	8046.7	2036.25	3.951	.9056	.3088	.5968	.3088	2036	0	0

FIGURE 11. *American Rowing Records.*

Time-distance to logarithmic co-ordinates.

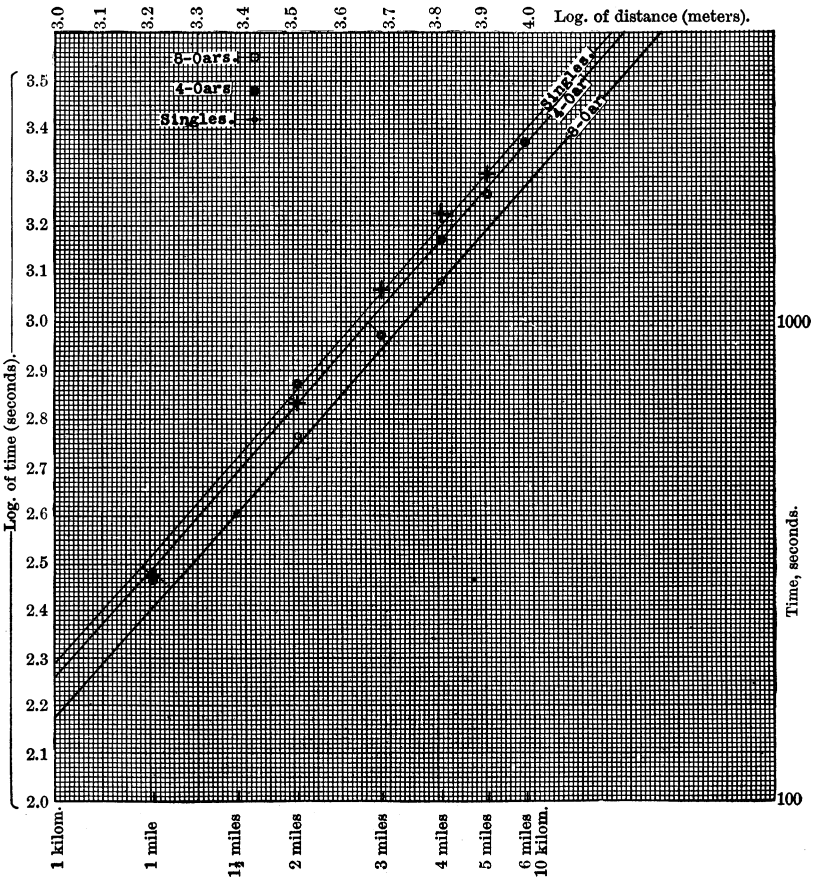


TABLE VII.

QUANTITATIVE RESULTS OF ANALYSIS OF ROWING RECORDS.

8-oars.	4-oars.	Singles.
$\log T = \frac{9}{8}\log L - 1.202$	$\log T = \frac{9}{8}\log L - 1.1145$	$\log T = \frac{9}{8}\log L - 1.085$
$\log V = 1.202 - \frac{1}{8}\log L$	$\log V = 1.1145 - \frac{1}{8}\log L$	$\log V = 1.085 - \frac{1}{8}\log L$
$T = \frac{L^{\frac{9}{8}}}{15.92}$ seconds	$T = \frac{L^{\frac{9}{8}}}{13.02}$ seconds	$T = \frac{L^{\frac{9}{8}}}{12.16}$ seconds
$V = \frac{15.92}{L^{\frac{1}{8}}} \frac{\text{meters}}{\text{second}}$	$V = \frac{13.02}{L^{\frac{1}{8}}} \frac{\text{meters}}{\text{second}}$	$V = \frac{12.16}{L^{\frac{1}{8}}} \frac{\text{meters}}{\text{second}}$

It is seen that for any given distance over 1 mile, the speeds of 8*s*, 4*s*, and singles are in the ratio 15.92 : 13.02 : 12.16, or 100 : 81.8 : 76.4, to the degree of approximation supported by the analysis.

MEN SWIMMING.

The data for swimming have been taken from the world's records given on page 252 of "The World Almanac" for 1905, setting forth 38 events from 25 yards to 4000 yards by amateurs and by professional swimmers of all nations. These data appear in columns I and II of Table VIII. The remaining columns work from these data as in preceding tables.

In Figure 12, log T' is plotted against log L, and also log V against log L. The ascending time-distance straight line is drawn through the 220-yard (201.2-meter) record point, and thence in a direction to conform fairly well with the other record points. It is drawn to make an angle of 48°.22′ or $\tan^{-1}\frac{9}{8}$ with the axis of distances. The descending straight line of speed-distance is likewise drawn through the 220-yard record point, and to make an angle of $-7° \; 7' \; 30''$ or $\tan^{-1} - \frac{1}{8}$ with the same axis.

Referring to the latter line, it will be observed that there is no visible retardation of speed over the short courses even down to 25 yards (22.9 meters). On the contrary, the speeds for the first four events, up to 60 yards (55 meters), inclusive, are higher than correspond to the logarithmic straight line. This may be accounted for by the fact that swimming speeds are only 15.5 per cent of men's running speeds at any given distance beyond 100 meters; so that the retardation at starting must be much less than in running. Moreover, it is possible that the

TABLE VIII.

Men Swimming.

Analysis of Best Professional and Amateur Records.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.
Distance, yards.	Time T, seconds.	Distance L, meters.	V, $\frac{\text{meters}}{\text{seconds}}$.	log T.	log L.	log V.	log T' computed.	T' computed seconds.	$T'—T$ deviation, seconds.	$100\left(\frac{T'-T}{T}\right)$, per cent deviation.
25	A 12.2	22.86	1.874	1.0864	1.3591	0.2727	1.1094	12.87	0.67	5.5
45	A 23.0	41.15	.789	.3617	.6144	.2527	.3966	24.93	1.93	8.4
50	A 24.6	45.72	.859	.3909	.6601	.2692	.4480	28.05	3.45	14.0
60	A 34.0	54.86	.614	.5315	.7393	.2078	.5371	34.44	0.44	1.3
80	P 51.0	73.15	.434	.7076	.8642	.1566	.6776	47.6	—3.4	—6.7
90	A 51.6	82.3	.595	.7126	.9154	.2028	.7352	54.35	2.75	5.3
100	A 58.0	91.44	.577	.7634	.9611	.1977	.7866	61.18	3.18	5.5
110	A 61.0	100.6	.649	.7853	2.0025	.2172	.8332	68.11	7.11	11.7
120	A 72.2	109.7	.520	.8585	.0403	.1818	.8751	75.01	2.81	3.9
150	A 96.6	137.2	.420	.9850	.1372	.1522	.9847	96.54	—0.06	—0.1
200	A 140.0	182.9	.306	2.1461	.2622	.1161	2.1254	133.5	—6.5	—4.6
220	A 148.6	201.2	.354	.1720	.3036	.1316	.1720	148.6	0	0
250	P 177.8	228.6	.285	.2499	.3591	.1092	.2344	171.5	—6.3	—3.5
300	A 214.6	274.3	.278	.3316	.4382	.1066	.3121	205.2	—9 4	—4.4
330	O 252.3	301.8	.196	.4019	.4797	.0778	.3700	234.5	—17.8	—7.1

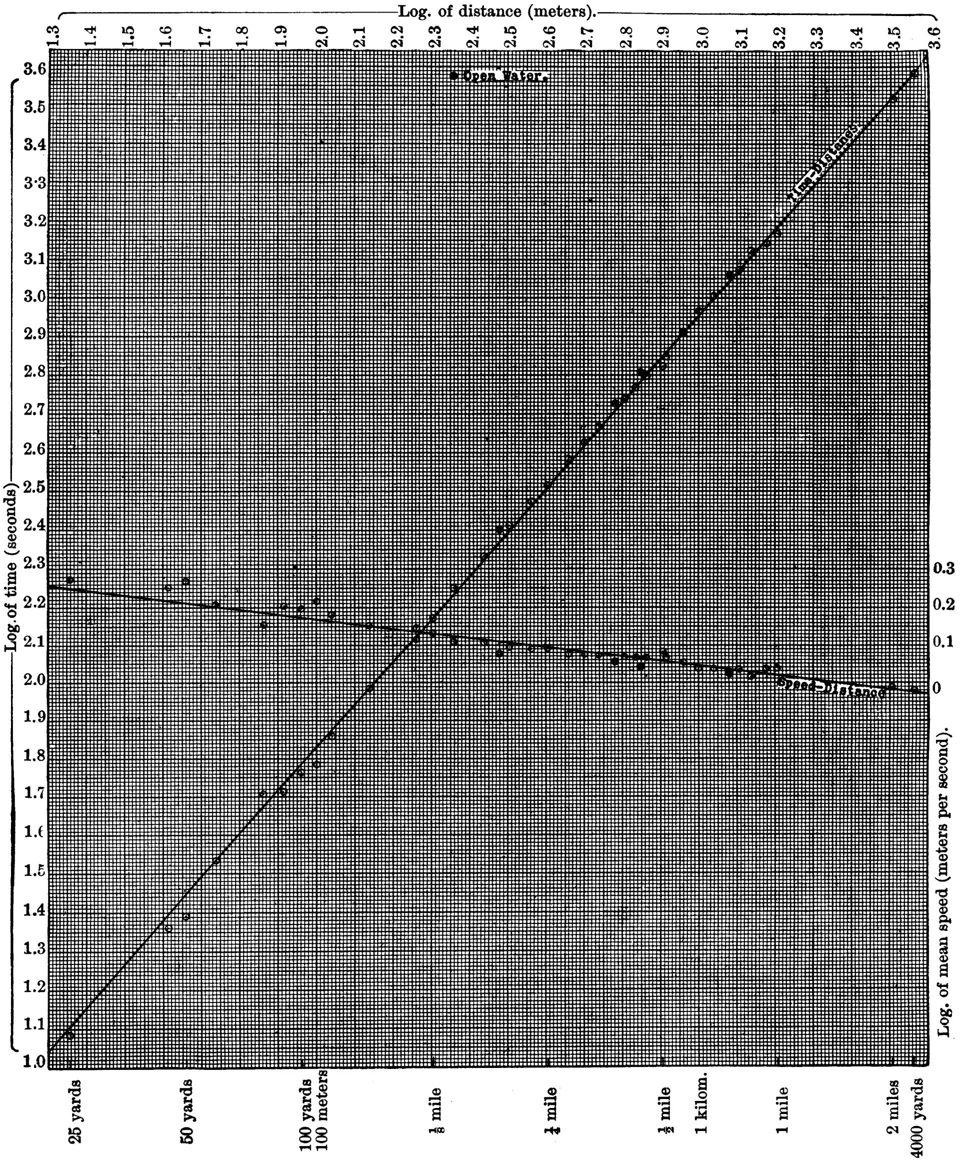
350	P 258.0	320.0	.240	.4116	.5052	.0936	.3988	250.5	–7.5	–2.9
400	A 297.0	365.7	.231	.4728	.5631	.0903	.4639	291.0	–6.0	–2.0
440	A 326.2	402.3	.233	.5135	.6046	.0911	.5106	324.0	–2.2	–0.7
500	A 383.6	457.2	.192	.5838	.6601	.0763	.5730	374.1	–9.5	–2.5
550	A 423.4	502.9	.188	.6268	.7015	.0747	.6196	416.5	–6.9	–1.6
600	A 465.0	548.6	.180	.6675	.7393	.0718	.6621	459.3	–5.7	–1.2
660	O 533.0	603.5	.132	.7267	.7807	.0540	.7087	511.3	–21.7	–4.1
700	A 547.4	640.1	.169	.7383	.8062	.0679	.7374	546.3	–1.1	–0.2
750	A 587.4	685.8	.167	.7689	.8362	.0673	.7711	590.4	3.0	0.5
770	O 642.0	704.1	.097	.8075	.8476	.0401	.7840	608.1	–33.9	–5.3
800	A 628.0	731.5	.165	.7980	.8642	.0662	.8026	634.8	6.8	1.1
880	P 664.5	804.6	.211	.8225	.9056	.0831	.8492	706.6	42.1	6.3
900	P 706.0	823.0	.166	.8488	.9154	.0666	.8602	724.8	18.8	2.7
1000	A 812.8	914.4	.125	.9100	.9611	.0511	.9116	815.8	3.0	0.4
1100	A 925.4	1005.8	.087	.9663	3.0025	.0362	.9582	908.2	–17.2	–1.9
1200	A 1011.0	1097.2	.085	3.0048	.0403	.0355	3.0007	1002.0	–9.0	–0.9
1320	O 1148.5	1207.0	.051	.0601	.0817	.0216	.0473	1115.0	–33.5	–2.9
1400	A 1183.0	1280.2	.082	.0729	.1073	.0344	.0761	1192.0	9.0	0.5
1500	A 1324.6	1371.6	.035	.1221	.1372	.0151	.1098	1288.0	–36.6	–2.8
1650	A 1390.4	1508.7	.085	.1431	.1786	.0355	.1563	1433.0	42.6	3.1
1760	A 1476.2	1609.3	.090	.1691	.2066	.0375	.1887	1544.0	67.8	4.6
3520	A 3294.0	3218.7	0.9772	.5177	.5076	1.9899	.5265	3362.0	68.0	2.1
4000	A 3824.0	3657.0	0.9564	.5825	.5631	.9806	.5889	3881.0	57.0	1.5

A denotes an amateur record. P, a professional record. O, a professional record in open water.

Except as above indicated, the records are stated to have been made in baths or bathing houses, as distinguished from open water.

FIGURE 12. *Swimming Records.*

Time-distance and speed-distance to logarithmic co-ordinates.



starting plunge made by the swimmer may actually advance him, relatively speaking, on the shortest courses.

With the exception of the four open-water events, on all of which the speed is distinctly low, and the 80-yard event, which has an unduly low speed (about the same as in the 150-yard event), the observations cling closely to the straight line. The speeds at both the half-mile and the 1-mile events appear to be distinctly higher than the rest.

Table VIII indicates that the mean deviation, without regard to sign, of all the events is 3.5 per cent.

The formulas pertaining to the straight lines in Figure 12 are

$$\log T = \tfrac{9}{8} \log L - 0.4196 \qquad (20)$$

$$\log V = 0.4196 - \tfrac{1}{8} \log L \qquad (21)$$

$$T = \frac{L^{\frac{9}{8}}}{2.628} \quad \text{seconds} \qquad (22)$$

$$V = \frac{2.628}{L^{\frac{1}{8}}} \quad \text{meters per second.} \qquad (23)$$

Men Skating.

Table IX gives the analysis of 24 yard and mile events and also 5 metric events, between the limits of 50 yards and 100 miles. The data appearing in columns I and II of this Table are taken from the records of the Amateur Athletic Union revised in 1905 and published on page 265 of "The World Almanac" for 1906. They represent, therefore, amateur records.

In Figure 13, $\log T$ has been plotted against $\log L$, crossing the sheet twice for economy in space. The straight line drawn in two sections is carried through the 1-mile record point. It also runs very near to the 2-mile, 4-mile, 500-meter, and 5000-meter points. There is a wide deviation of the points from the line between 50 yards (45.72 meters) and 440 yards (402.3 meters). This may be attributed to the influence of inertia in acceleration at the start. Column IV of Table IX shows in fact that the maximum speed over a course is not reached until the 440-yard event (402.3 meters), when it attains 11.43 meters per second (25.5 miles per hour). The entries in column IV also reveal considerable relative variation, and do not descend with the same degree of uniformity as is manifested by the corresponding speed entries in Table VIII, or other tables. Perhaps this variability in the

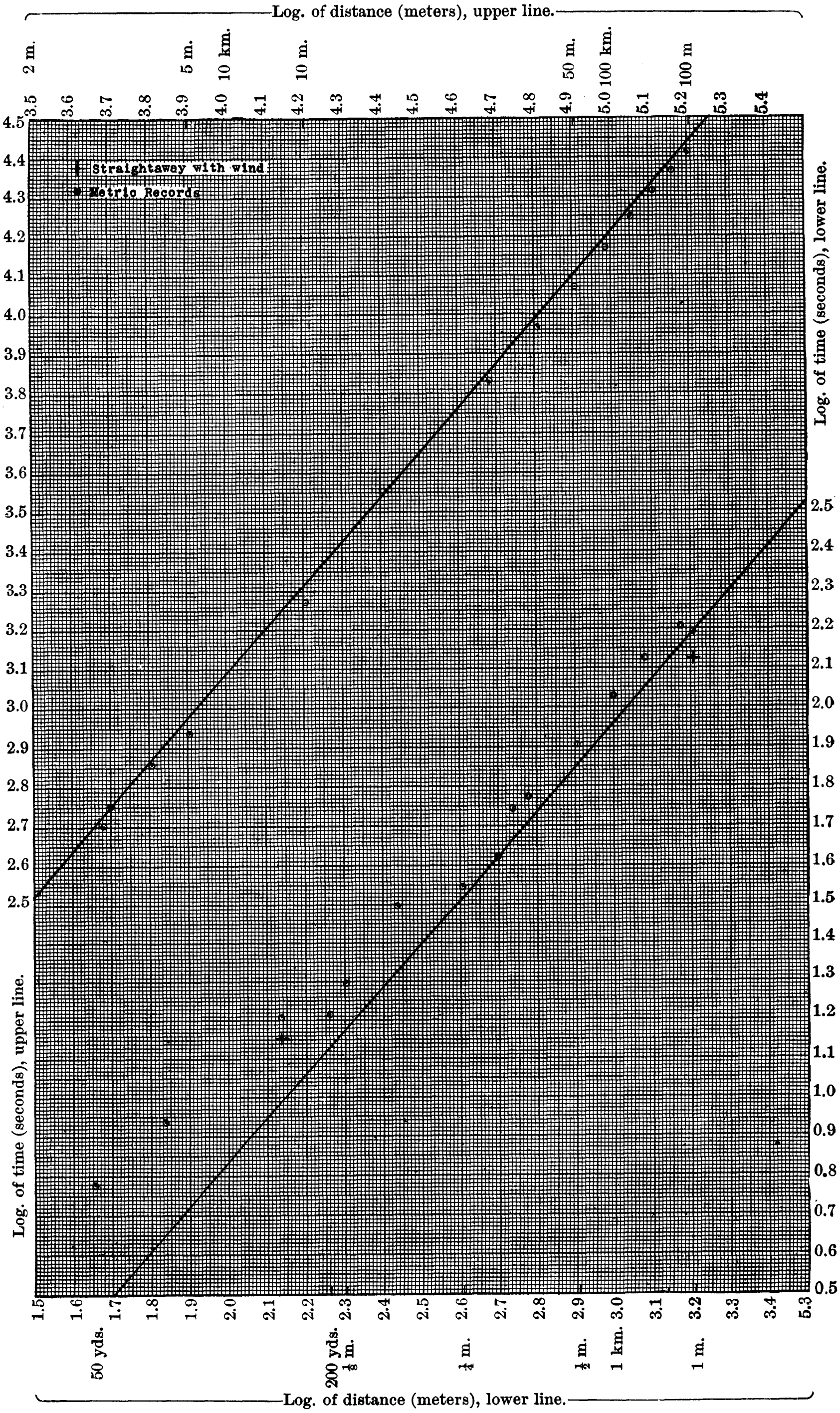
TABLE IX. — MEN SKATING.

Analysis of Amateur Skating Records.

I.	II.	III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.
Distance, yards.	Time T, seconds.	Distance L, meters.	Speed V, $\frac{\text{meters}}{\text{seconds}}$.	log L.	log T' computed.	log T.	log V.	T' seconds, computed.	$T'-T$ deviation, seconds.	$100\left(\frac{T'-T}{T}\right)$ deviation per cent.	Mean percentage deviation.
50	6.0	45.72	7.62	1.6601	0.4533	0.7782	0.8819	2.84	–3.16	–52.0	
75	8.6	68.58	7.974	.8362	.6514	.9345	.9017	4.48	–4.12	–48.0	
150	15.88	137.16	8.635	2.1372	.9901	1.2009	.9363	9.78	–6.1	–38.3	
200	16.4	182.88	11.150	2622	1.1307	.2148	1.0474	13.51	–2.9	–17.8	
220	19.8	201.2	10.160	.3036	.1773	.2967	.0069	15.04	–4.8	–24.0	
300	31.4	274.3	8.736	.4382	.3287	.4969	0.9413	21.32	–10.1	–32.1	
440	35.2	402.3	11.43	.6046	.5159	.5465	1.0581	32.8	–2.4	–6.8	
600	55.25	548.6	9.929	.7393	.6674	.7423	0.9970	46.5	–8.8	–15.8	7.6
880	80.4	804.6	10.01	.9056	.8545	.9053	1.0003	71.5	–8.9	–11.1	
1,320	133.0	1,207.0	9.075	3.0817	2.0526	2.1239	0.9578	112.9	–20.1	–15.1	
miles											
1	156.0	1,609.3	10.32	.2066	.1931	.1931	1.0135	156.0	0	0	
2	342.6	3,218.7	9.395	.5077	.5319	.5348	0.9729	340.3	–2.3	–0.7	
3	503.0	4,828.0	9.598	.6838	.7300	.7016	.9822	537.1	34.1	6.8	
4	720.5	6,437.4	8.934	.8087	.8705	.8576	.9511	742.2	21.7	3.0	
5	864.0	8,046.7	9.314	.9056	.9795	.9365	.9691	954.0	90.0	10.4	
10	1,871.2	16,093.0	8.600	4.2066	3.3181	3.2721	.9345	2,080.0	208.8	11.1	
30	6,800.0	48,280.0	7.100	.6838	.8550	.8325	.8513	7,161.0	361.0	5.3	
40	9,286.0	64,374.0	6.932	.8087	.9955	.9678	.8409	9,897.0	611.0	6.6	
50	11,759.4	80,467.0	6.842	.9056	4.1045	4.0704	.8352	12,720.0	960.0	8.2	
60	14,820.6	96,561.0	6.515	.9848	.1936	.1709	.8139	15,620.0	800.0	5.4	
70	17,715.6	112,654.0	6.358	5.0517	.2689	.2483	.8034	18,570.0	854.0	4.8	
80	20,515.0	128,748.0	6.275	.1097	.3341	.3121	.7976	21,580.0	1,065.0	5.2	
90	23,157.6	144,841.0	6.254	.1608	.3916	.3647	.7961	24,640.0	1,482.0	6.4	
100	25,898.0	160,935.0	6.213	.2066	.4531	.4133	.7933	28,390.0	2,492.0	9.6	
meters											
500	41.8	500.0	11.95	2.6990	1.6221	1.6218	1.0772	41.9	0.1	0.2	
600	59.6	600.0	10.07	.7782	.7112	.7752	1.0030	51.4	–8.2	–13.8	
1,000	107.0	1,000.0	9.346	3.0000	.9607	2.0294	0.9706	91.4	–15.6	–14.6	
1,500	160.8	1,500.0	9.328	.1761	2.1588	.2063	0.9698	144.1	–16.7	–10.4	
5,000	565.4	5,000.0	8.843	.6990	.7471	.7524	0.9466	558.6	–6.8	–1.2	

FIGURE 13. *Amateur Skating Records.*

Time-distance to logarithmic co-ordinates.



records is due in some measure to the influence of wind, which is particularly active on skating speeds.

Column XII of Table IX indicates that the mean percentage deviation of the series of computed times against record times is 7.6 per cent between 500 meters and 100 miles (160.9 kilometers). If the same test is applied to the entire series between 50 yards and 100 miles, the mean percentage of deviation, without regard to sign, is 13.3 per cent.

The following formulas pertain to the straight line of Figure 13 :

$$\log T = \tfrac{9}{8} \log L - 1.4143 \qquad (24)$$

$$\log V = 1.4143 - \tfrac{1}{8} \log L \qquad (25)$$

or

$$T = \frac{L^{\frac{9}{8}}}{25.96} \text{ seconds} \qquad (26)$$

$$V = \frac{25.96}{L^{\frac{1}{8}}} \text{ meters per second.} \qquad (27)$$

MEN BICYCLING.

The data for bicycling have been taken from pages 267 and 268 of "The World Almanac" for 1906, and are embodied in columns I and II of Table X. The first five events are professional paced records against time. The next series of events, from 2 miles to 100 miles, inclusive, are professional motor-paced records, in competition. The remainder are stated to be American competition, professional, paced, hour records. The average speeds over the distances are set down in column V. It will be seen that the speed between 2 miles (3.22 kilometers) and 30 miles (48.28 kilometers) was almost precisely uniform at 23.5 meters per second. In fact, it appears from the table in "The World Almanac" that all of these records were made by one and the same individual, on one and the same day (May 31, 1904), at Charles River Park, Mass.. Again, from 31 miles (49.89 kilometers) to 50 miles (59.59 kilometers), the speeds are nearly uniform at 22.8 meters per second (51 miles per hour), and these appear to have been likewise made at Charles River Park by the same rider on the same day (September 1, 1903). There is very little fall of speed between the 50-mile event and the 2-mile event, or apparently but little fatigue as far as 50 miles. Beyond 50 miles, however, the speeds fall off and fatigue is indicated.

TABLE X.

Men Bicycling.

Analysis of Professional Bicycling Paced Records.

I.	II.	III.	IV.	V.
Distance, miles.	Time, h. m. s.	Distance L, meters.	Time T, seconds.	Speed V, $\frac{\text{meters}}{\text{seconds}}$.
0.25	20.0	402.3	20.0	20.12
0.33	27.8	536.4	27.8	19.3
0.5	41.0	804.7	41.0	16.92
0.66	58.6	1,072.8	58.6	18.30
1	1 06.2	1,609.3	66.2	24.30
2	2 19.0	3,218.7	139.0	23.16
3	3 31.6	4,828.0	211.6	22.82
4	4 43.0	6,437.4	283.0	22.75
5	5 51.0	8,046.7	351.0	22.93
6	7 00.2	9,656.1	420.2	22.98
7	8 07.6	11,265.4	487.6	23.10
8	9 14.2	12,874.8	554.2	23.23
9	10 22.0	14,484.1	622.0	23.28
10	11 29.2	16,093.0	689.2	23.35
11	12 36.2	17,703.0	756.2	22.88
12	13 43.0	19,312.0	823.0	23.46
13	14 50.4	20,921.0	890.4	23.50
14	15 57.2	22,531.0	957.2	23.54
15	17 03.4	24,140.0	1,005.4	24.00
16	18 10.6	25,749.0	1,090.6	23.61
17	19 17.4	27,359.0	1,157.4	23.64
18	20 24.2	28,968.0	1,224.2	23.66
19	21 30.8	30,577.0	1,290.8	23.68
20	22 37.6	32,187.0	1,357.6	23.71
21	23 44.6	33,796.0	1,424.6	23.72
22	24 51.8	35,406.0	1,491.8	23.73
23	25 59.0	37,015.0	1,559.0	23.75
24	27 07.6	38,624.0	1,627.6	23.73
25	28 14.2	40,234.0	1,694.2	23.75
26	29 22.6	41,843.0	1,762.6	23.73
27	30 30.2	43,452.0	1,830.2	23.74
28	31 37.4	45,062.0	1,897.4	23.75
29	32 48.0	46,671.0	1,968.0	23.72
30	33 52.6	48,280.0	2,032.6	23.77
31	36 26.0	49,890.0	2,186.0	22.82
32	37 37.2	51,499.0	2,257.2	22.81
33	38 48.8	53,109.0	2,328.8	22.81
34	39 57.6	54,718.0	2,397.6	22.82
35	41 07.6	56,327.0	2,467.6	22.82
36	42 18.2	57,937.0	2,538.2	22.83
37	43 28.2	59,546.0	2,608.2	22.83
38	44 39.2	61,155.0	2,679.2	22.82
39	45 49.4	62,765.0	2,749.4	22.83
40	47 00.0	64,374.0	2,820.0	22.83

TABLE X. — *Continued.*

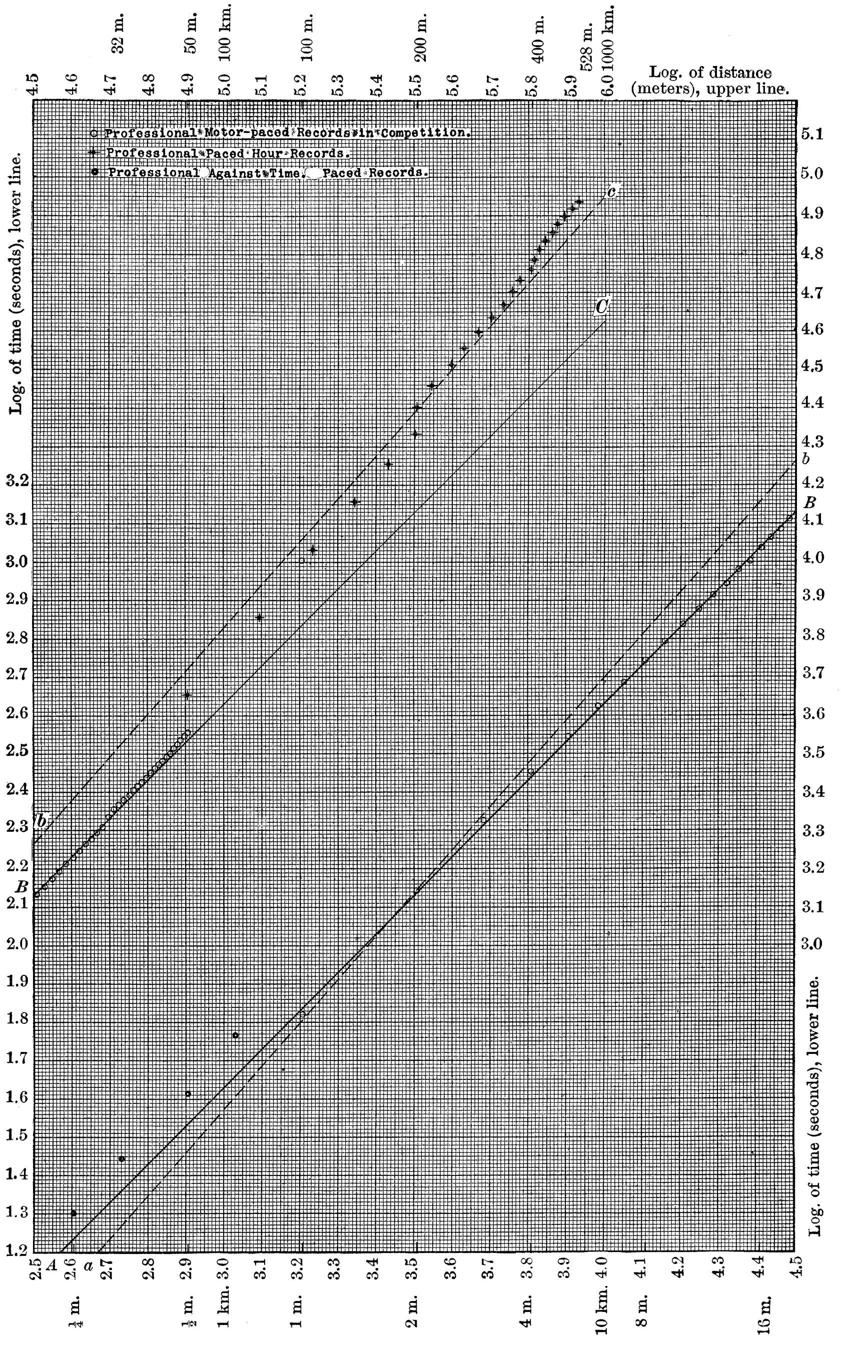
I.	II.	III.	IV.	V.
Distance, miles.	Time, m. h. s.	Distance L, meters.	Time T, seconds.	Speed V, $\frac{\text{meters}}{\text{seconds}}$.
41	48 10.8	65,983.0	2,890.8	22.82
42	49 21.2	67,593.0	2,961.2	22.82
43	50 31.2	69,202.0	3,031.2	22.83
44	51 41.2	70,811.0	3,101.2	22.83
45	52 20.8	72,421.0	3,170.8	22.84
46	54 23.8	74,030.0	3,263.8	22.68
47	55 49.6	75,639.0	3,349.6	22.58
48	57 21.2	77,249.0	3,441.2	22.45
49	58 43.2	78,858.0	3,523.2	22.38
50	59 59	80,467.0	3,599.0	22.36
100	2 48 11.8	160,930.0	10,091.8	15.58
miles yds.				
50 3	1	80,470.0	3,600.0	22.35
77 440	2	124,322.0	7,200.0	17.27
106 900	3	171,409.0	10,800.0	15.87
137 275	4	220,728.0	14,400.0	15.33
168 910	5	271,198.0	18,000.0	15.07
197 220	6	317,241.0	21,600.0	14.69
199 220	7	320,456.0	25,200.0	12.71
218 440	8	351,230.0	28,800.0	12.20
246 440	9	396,300 0	32,400.0	12.23
265	10	426,480.0	36,000.0	11.84
289	11	465,100.0	39,600.0	11.75
312 880	12	502,920.0	43,200.0	11.64
335 1540	13	540,535.0	46,800.0	11.55
355	14	571,314.0	50,400.0	11.33
372	15	598,670.0	54,000.0	11.09
397 220	16	639,100.0	57,600.0	11.09
403 440	17	649,000.0	61,200.0	10.60
416	18	669,500.0	64,800.0	10.33
432	19	695,200.0	68,400.0	10.16
450 1540	20	725,600.0	72,000.0	10.08
466 660	21	750,600.0	75,600.0	9.928
485 220	22	780,700.0	79,200.0	9.858
507 1320	23	817,100.0	82,800.0	9.870
528 925	24	850,600.0	86,400.0	9.846

Figure 14 shows log T plotted against log L, as in preceding cases. The observations fall near to a straight line, A B, B C, as far as 50 miles. This line makes an angle of 45° with the axis of distances. For the purposes of comparison a broken line *ab*, *bc* is drawn through the 2-mile record point at an angle of 48° 22′ to correspond with the lines in all of the other figures. The contrast is very noticeable. Beyond 100 miles (160.9 kilometers) the points tend to follow the direction of the broken line, but not with any degree of precision.

The inference to be drawn from Table X and Figure 14 is that

FIGURE 14. *Bicycling Records.*

Time-distance to logarithmic co-ordinates.



we have no means of discovering from these records what the highest speed of a bicycle rider may be. It is inconceivable that there should be no fatigue for 50 or 30 miles. If the record speed over 30 miles be 23.77 meters per second, the speed over 2 miles should be much greater. If the same law of fatigue held for bicycle riders as for runners, walkers, swimmers, and skaters, the speed at 2 miles should be approximately $\sqrt[8]{15}$ or 1.40 times greater; viz., 33.3 meters per second, or 74.4 miles per hour. There is no proof, however, that the same law of fatigue applies, and the air resistance at such high speeds might influence the results. It is, however, evident that the speed at 2 miles, or similar distances, is kept down abnormally to that at 30 miles. The explanation suggests itself that the records are all made on a circular track of considerable lateral inclination. The cyclist, on short runs, perhaps attains the highest speed that he dares and not the highest speed that his muscles could develop. When travelling at 23.75 meters per second (53 miles per hour), careful steering must be needed to keep on the track, and perhaps the records indicate the limit of steering nerve rather than the limits of speed and endurance below 30 or 50 miles. The case is somewhat similar to that of automobiles in this respect. The track records of heavy-weight gasolene automobiles, as given in "The World Almanac" for 1906, indicate speeds of 30 meters per second (67 miles per hour) at 1 mile, and hardly any reduction up to 10 miles, or no sensible fatigue within those limits. At 1000 miles (1609 kilometers) the speed is 20.35 meters per second (45.5 miles per hour). But on the straightaway courses, as distinguished from track courses, the speed averaged 46.77 meters per second (104.5 miles per hour) at 1 mile, and fell off distinctly with distance at a rate very similar to the fatigue rate of racing animals. Until, therefore, we have a wide straightaway course, say 5 kilometers long, provided for cyclists, of as good quality throughout as is presented in circular tracks, the cyclist's maximum speed will remain a matter of doubt.

Summary of Results.

A summary of the results of the various analyses in regard to accuracy is presented in Table XI. Column I refers to the table considered. Column II, the character of the race. The total number of records in each table appears in column III. The range of distances covered is given in column IV, both in miles and in kilometers. The sum total of the percentage deviations, without regard to their sign, as found for each table, including every record, is given in column V. The quotient of the sum in column V, by the number of records in

TABLE XI.

Summary of Deviations.

I.	II.	III.	IV.		V.	VI.	VII.	VIII.		IX.	X.
Table.	Racers.	Total number of Records.	Range of Distances.		Sum total of percentage Deviations without regard to Sign.	Average percentage Deviation over entire series of Records.	Number of properly acceptable Records.	Range of Distances.		Total percentage Deviations.	Average percentage Deviations.
			Miles.	Kilometers.				Miles.	Kilometers.		
I	Horses trotting	10	1–100	1.61–161	92.9	9.29	7	1.00–20	1.61–32.2	12.9	1.84
II	Horses running	33	$\frac{1}{4}$–4	0.40–6.4	80.3	2.43	28	0.63–4	1.00–6.4	53.1	1.89
III	Horses pacing	6	$\frac{1}{2}$–5	0.80–8.0	32.4	5.40	3	$\frac{1}{2}$–2	0.80–3.2	5.7	1.90
IV	Men running	92	$\frac{1}{80}$–623	0.02–1000	807.1	8.77	69	$\frac{1}{22}$–60	0.07–96.5	286.3	4.15
V	Men walking	32	1–531	1.60–855	178.8	5.59	25	1.00–120	1.60–193	83.7	3.35
VI	Men rowing	17	$\frac{1}{4}$–6	0.40–9.7	101.5	5.97	15	1.00–6	1.60–9.7	65.5	4.36
VIII	Men swimming	38	$\frac{1}{70}$–2.3	0.02–3.7	133.8	3.52	38	$\frac{1}{70}$–2.3	0.02–3.7	133.8	3.52
IX	Men skating	29	$\frac{1}{35}$–100	0.05–161	384.7	13.26	22	$\frac{1}{3}$–100	0.55–161	165.7	7.53
		257			1811.5	7.05	207			806.7	3.9

column III, gives the mean percentage deviation for each table in column VI, or the mean difference between the computed record time and the actual record time in percentage of the latter. Reasons have been given in connection with each table why certain records should be left out of consideration. The remainder are discussed in columns VII, VIII, IX, and X. These contain what may be called the net results, while columns III, IV, V, and VI contain the gross results.

In the net results, horses come out the best, and nearly equally well for trotting, running, or pacing, viz., 1.9 per cent mean deviation. The horses come out much better than the men in this comparison. It should be observed, however, that the ranges of distances covered by the horse-races are relatively small, — 20, 6.4, and 4, — whereas with men the ranges are successively 1320, 120, 6, 185, 300, — much greater. Perhaps if the ranges covered in the horses' performances had been similar to those covered in the men's performances, the disparity in precision would disappear. In the men's performances the net average deviation is about 4 per cent, except in skating, where it is 7.5 per cent. The net average deviation of all the 207 records is 3.9 per cent.

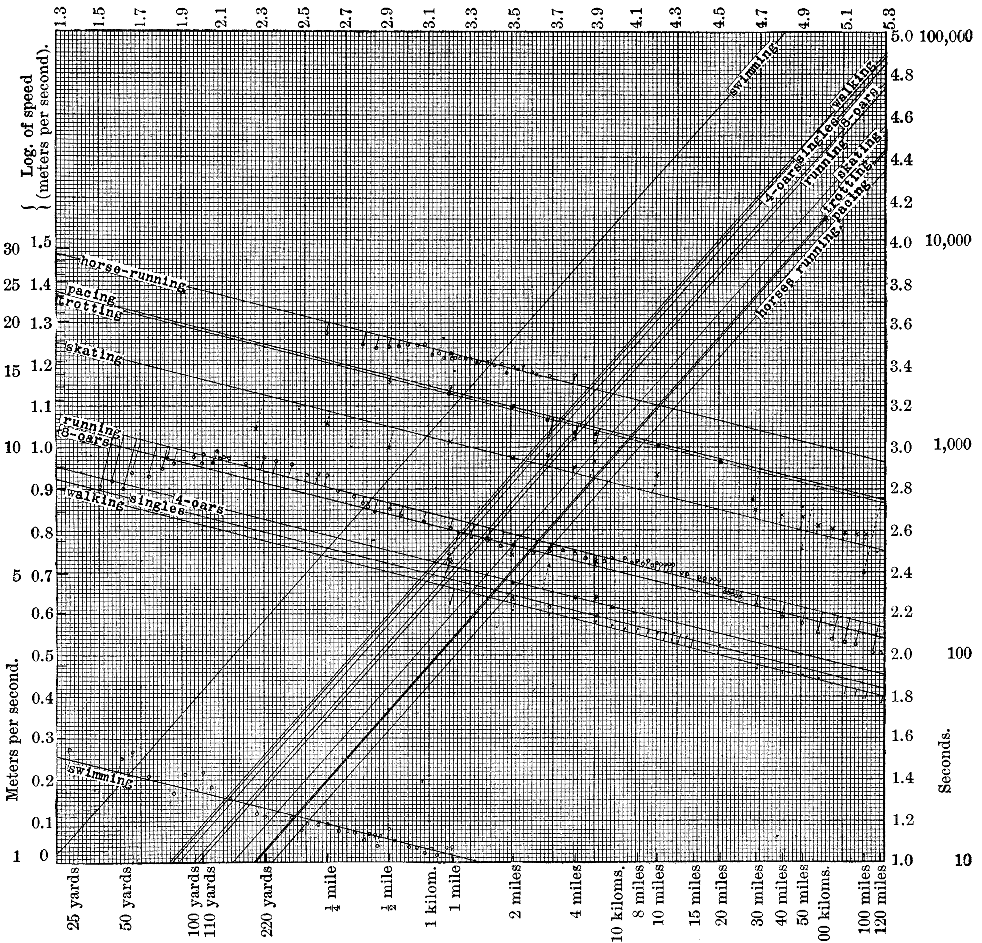
Considering the gross results of columns III, IV, V, and VI, the lowest deviation is found in trotting horses (2.43 per cent) followed by men swimming (3.52 per cent). The greatest deviation is in skating (13.26 per cent). The mean deviation of the whole series of 257 records, rejecting none, is 7.05 per cent.

It is submitted that the summary in Table XI demonstrates the proposition that the records in races of men and of horses approximately follow straight lines when plotted on logarithm paper; because the average percentage deviation of all the records is only 7 per cent from the line, and excluding 50 of the records as unreliable for reasons assigned, this average deviation falls to 4 per cent. The record time that should belong to any given distance within the usual limits, and for any of the events considered, except bicycling, can thus be assigned with these probable degrees of accuracy.

It is not so remarkable that the records of any one event, such as men running, should approximately conform to a logarithmic straight line; but it is remarkable that the straight lines should be parallel, or substantially parallel, in all of these eight classes of events, including three gaits in quadrupeds and three gaits in bipeds, besides motion in the water and over ice. Figure 15 collects all of the logarithmic straight lines on one sheet to a reduced scale. The ascending parallel straight lines are time-distance lines. The descending parallel straight

FIGURE 15. *Speed-distance and Time-distance Lines.*

Ascending lines, time-distance ; descending lines, speed-distance.



EXPLANATION. — At the right of the Figure the numerals 1.0 to 5.0 are logarithms of time (seconds).

lines are speed-distance lines. The mean speeds of the various events in Table I to IX inclusive are plotted with relation to the descending lines in such a manner as to reveal their deviations as clearly as possible.

DEDUCTIONS FROM THE RESULTS.

Starting with the approximate equation

$$T \cong \frac{L^{\frac{9}{8}}}{c} \text{ seconds} \qquad (28)$$

as borne out in the preceding discussions of records in eight different kinds of races, where c is a constant for each type of race, it follows that as the length of the course, L, increases, the time occupied in the race will increase according to the following table :

TABLE XII.

EFFECT OF INCREASE OF DISTANCE UPON INCREASE IN TIME.

Increase in Distance.	Increase in Time.	Increase in Distance.	Increase in Time.
2	2.181	70	119.1
3	3.442	80	138.4
4	4.757	90	158.0
5	6.114	100	177.8
6	7.506	200	387.8
7	8.928	300	770.5
8	10.38	400	845.9
9	11.85	500	1087.0
10	13.33	600	1335.0
20	29.08	700	1588.0
30	45.90	800	1845.0
40	63.43	900	2106.0
50	81.54	1000	2371.0
60	100.1		

Thus, if the equation (28) is correct, doubling the distance means increasing the time by 118 per cent, and increasing the distance 60 times increases the time 100 times. Since the equation (28) is not set up as a rigid law, but as a statistical approximation, or approximative law, we cannot expect to find the relations of Table XII accurately presented by all the events. For instance, if we limit ourselves to the first statement that twice the distance should be covered in 2.18 times the time, we can examine all the cases of pairs of such events in the records already considered and find what the ratios of time are when the distances are as 2 : 1. The answer to this question is contained in the following table:

TABLE XIII.

Analysis of Time Ratios over Pairs of Courses in Ratio of 2 : 1.

Table.	Type of Racers.	No. of Pairs.	Range of Distances covered.		Sum of Ratios.	Mean Ratio.	Highest Ratio.	Lowest Ratio.
			Miles.	Kilometers.				
I	Horses trotting	5	1–100	1.61–161	11.092	2.218	2.327	2.098
II	Horses running	19	0.25–4.0	0.4–6.4	40.523	2.133	2.244	2.000
III	Horses pacing	3	0.5–4	0.8–8.0	6.662	2.221	2.374	2.058
IV	Men running	57	0.03–300	0.02–483	123.754	2.171	2.593	1.818
V	Men walking	18	1–500	1.61–805	39.678	2.204	2.375	2.073
VI	Men rowing	8	0.25–6	0.4–9.6	17.950	2.244	2.518	1.981
VIII	Men swimming	23	0.014–2	0.023–3.2	50.337	2.189	2.436	2.016
IX	Men skating	13	0.03–100	0.05–161	27.198	2.092	2.556	1.760
	General average	146	. . .	. . .	317.194	2.173	2.593	1.760

Taking, for example, the case of running men with reference to Tables IV and XIII, there are 57 pairs of records in which one distance is just double the other, commencing at 20–40 yards and ending with 150–300 miles. The sums of all the time ratios for these 57 pairs of events is 123.754, representing an average value of 2.171, as against 2.181 required by Table XII from equation (28). The highest ratio in any of these pairs is 2.593, and the lowest 1.818.

It will be seen that men running and men swimming come nearest in their averages to the ratio 2.181; while men skating deviate the most (4 per cent). Taking all the 146 pairs of double distances presented in the entire set of records, and without rejecting any, the average ratio of all is 2.173, which is within 0.4 per cent of the value 2.181 required in equation (28). This result constitutes an independent demonstration of the approximate accuracy of that equation. If pairs at short distances affected by starting retardation, or at very long distances, had been rejected, the agreement would have been still closer.

Equation (28) leads to the following :

$$T \cong \frac{c^8}{V^9} \text{ seconds} \tag{29}$$

or the time varies approximately inversely as the ninth power of the speed in the race. If we assume that the racer reaches the winning post virtually exhausted, so far as affects racing effort, then the time of exhaustion varies inversely as the ninth power of the speed. The computed effect of increasing speed is given in the following table :

TABLE XIV.

COMPUTED INFLUENCE OF SPEED UPON THE TIME OF EXHAUSTION.

Speed.	Time.	Reciprocal.	Speed.	Time.	Reciprocal.
1.0	1.000	1.0	1.5	0.0260	38.4
1.01	0.915	1.09	1.6	0.0146	68.7
1.1	0.424	2.36	1.7	0.00843	119
1.2	0.194	5.16	1.8	0.00504	198
1.3	0.0943	10.6	1.9	0.00310	322
1.4	0.0484	20.7	2.0	0.00195	512

Table XIV shows that if a racer increases his speed 10 per cent, he brings down his running time from 100 to 42.4, or 2.36 times; and if he doubles his speed, he becomes exhausted in 0.195 per cent of the original time, or 512 times more quickly.

The records do not show that any one racer would be exhausted 512 times as soon if he doubled his speed, because there is no evidence at hand as to the behavior of any single individual at doubled speeds. If, however, any one racer could be trained to take the record speed at each event in the entire series, — that is to say, if a runner could be trained to take every race from the 20-yard dash to the 50-mile run at world's record speeds in each, — then this ideal athlete would be exhausted in a time approximately as the inverse ninth power of the speed. It is reasonable to assume that what would be true of this ideal athlete also tends to be true of any normal athlete, even though his range of speeds and distances be limited.

As an example of this rapid rate of exhaustion, take the 20-mile (32.19 kilometers) running event in Table IV. The speed at which this race was run averaged 4.794 meters per second as shown in column IV. At twice this speed or 9.588 meters per second, there is no exact distance in the table; but the nearest is 131.5 yards (120.2 meters) at 9.696 meters per second. The time of the 20-mile event was 6714 seconds in column II. The time of the 131.5 yard event was 12.4 seconds, or 541 times less, as against 512 times in Table XIV.

Again, consider the last event in Table VIII of swimming records. The 4000-yard (3657-meter) event was finished in 3824 seconds at a speed over the course of 0.9564 meters per second. If we take a speed 1.5 times greater than this, or 1.4346 meters per second, we find one near to it in the table; namely, 1.420 meters per second (column XI) in the 150-yard (137-meter) event. According to Table XIV, the time of exhaustion at 1.5 times greater speed is 38.4 times less than the original. The time should therefore be $3824 \div 38.4 = 99.6$ seconds. The actual time of the event is given as 96.6 seconds.

DISTANCE

Equation (28) leads to the following expression for L:

$$L \cong c^{\frac{8}{9}}\ T^{\frac{8}{9}} \quad \text{meters.} \tag{30}$$

That is, as more and more time is allowed for racers to occupy in an event, the distances they will traverse will not be directly proportional to the time, but will vary as the eighth power of the ninth root of the time, approximately. A few numerical values are given in the accompanying table:

TABLE XV.

DISTANCES TRAVERSED WITH INCREASING RACING TIME.

Time.	Distance.	Time.	Distance.
1	1	20	14.3
2	1.85	50	32.4
3	2.66	100	60.0
4	3.43	200	111.0
5	4.18	500	251.0
10	7.74	1000	464.0

It is thus indicated that with 500 times more time, the distance covered will be only 251 times greater. As an example, we may take the 4-mile (6.44 kilometers) walking event of table V. It occupied 1658 seconds. If we increase the time twenty times, or to 33,160 seconds, we should expect from Table XV that the distance covered would be 57.2 miles (92 kilometers). The nearest event to this in Table V is the 60-miles (96.6 kilometers), occupying 34,847 seconds or 21 times the original time, which is a satisfactory agreement.

Another consequence of equation (28) is expressed :

$$L \cong \frac{c^8}{V^8} \text{ meters} \qquad (31)$$

or the distance covered in a race varies approximately as the inverse eighth power of the speed adopted. That is, if an athlete could be trained to take any distance from the shortest to the longest at the record speed for the event; then the distance which this athlete would be able to run before being exhausted would be as the inverse eighth power of his speed over the course. A few numerical values are given in the accompanying table :

TABLE XVI.

DISTANCES CAPABLE OF BEING TRAVERSED AS THE SPEED IS INCREASED.

Speed, V.	Distance, V^{-8}.	Reciprocal, V^8.	Speed, V.	Distance, V^{-8}.	Reciprocal, V^8.
1.0	1.	1.	1.5	0.0390	25.6
1.01	0.923	1.08	1.6	0.0233	43.0
1.1	0.467	2.14	1.7	0.0143	69.8
1.2	0.233	4.30	1.8	0.00907	110.0
1.3	0.123	8.16	1.9	0.00589	170.0
1.4	0.0678	14.8	2.0	0.00391	256.0

The table shows that if the speed is doubled, the distance that can be run, before exhaustion, is reduced 256 times according to (31).

As an instance, the 20-mile (32.19 kilometers) running event of Table IV, already referred to, may be selected. The speed over the course was 4.794 meters per second. At 9.588 meters per second the distance should be $\frac{20}{256} = 0.0781$ mile $= 125.7$ meters. Table IV shows that at 9.696 meters per second, the nearest to the required speed, the distance run was 131.5 yards, or 120.2 meters.

Speed.

Equation (28) leads to the expression for speed:

$$V \cong c L^{-\frac{1}{8}} \cong \frac{c}{L^{\frac{1}{8}}} \text{ meters per second} \qquad (32)$$

already illustrated in formulas (3), (7), (11), (15), and (19). It means that the speed of racing over courses of different lengths varies inversely as the eighth root of the length, approximately. Table XVII gives a few numerical applications of this rule.

TABLE XVII.

Effect of increasing Distances upon the Speed over the Course.

Distance, L.	Speed, $L^{-\frac{1}{8}}$.	Reciprocal, $L^{\frac{1}{8}}$.	Distance, L.	Speed, $L^{-\frac{1}{8}}$.	Reciprocal, $L^{\frac{1}{8}}$.
1	1.	1.	100	0.562	1.78
2	0.917	1.09	200	0.516	1.94
3	0.872	1.15	300	0.490	2.04
4	0.840	1.19	400	0.473	2.12
5	0.815	1.22	500	0.460	2.17
6	0.799	1.25	600	0.449	2.23
7	0.784	1.28	700	0.441	2.27
8	0.771	1.30	800	0.433	2.31
9	0.760	1.32	900	0.427	2.34
10	0.750	1.33	1000	0.422	2.37
20	0.688	1.45	2000	0.387	2.59
30	0.654	1.53	3000	0.368	2.72
40	0.631	1.59	4000	0.355	2.82
50	0.613	1.63	5000	0.345	2.90
60	0.599	1.67	6000	0.337	2.97
70	0.588	1.70	7000	0.331	3.02
80	0.578	1.73	8000	0.325	3.08
90	0.570	1.75	9000	0.320	3.12

According to these results the speed has to be reduced 1.33 times to race 10 times the distance, and 1.78 times to race 100 times the distance. Thus, taking the 100.6-meter event (110 yards) in Table IV, the speed over the course is 9.144 meters per second. At 96.56 kilometers (60 miles), nearly 1000 times greater distance, the speed has fallen to 3.572 meters per second, or to 39.1 per cent of the former. According to Table XVII, the speed should fall to 42.2 per cent on increasing the distance 1000-fold.

Another consequence of formula (28) is expressed thus :

$$V \cong c^{\frac{8}{9}} T^{-\frac{1}{9}} \cong \frac{c^{\frac{8}{9}}}{T^{\frac{1}{9}}} \quad \text{meters per second} \qquad (33)$$

or the speed over the course approximately varies inversely as the ninth root of the racing time.

In Table XVIII the speeds of the various racers are set down as computed for courses of 1 kilometer and of 1 mile. Thus over 1-kilometer courses the speed of the running horse is 17.88 meters per second (column VI), and the time for the race 55.9 seconds. At the end of the series come swimmers, with a speed of 1.108 meters per second and an inferred kilometer-time of 902.4 seconds. Table VIII shows that the time for 1.006 kilometers was 925.4 seconds. Turning to the mile range, the speed of the running horse is 37.7 miles per hour over the 1-mile range. His mile-time is 95.5 seconds, both computed and recorded. The speed of the swimmer is 2.336 miles per hour, and the mile-time 1544 seconds as computed, and 1476.2 seconds as observed. The speed of a running man is almost precisely half that of the trotting horse, for distances above 1 kilometer where starting retardation ceases to affect the horse. The speed of a professional walker is very nearly the speed of a professional rower (singles).

It is to be noted that all these speeds are average speeds over the courses. There is no evidence among the records to show what the speed was at different points in the course. So far as concerns anything appearing in the data, the speed of a runner, for example, which averages 7.17 meters per second over a 1-kilometer course, might be 10 meters per second in the first part and 5 in the last part, or *vice versa*. Evidence is lacking to show what the facts are, and they are of great importance to the science of athletics. The speed of a world's-record type of trained runner might be determined at any or all points of a course, either by securing a light recording chronograph on the back of his belt, with a thread payed out as he ran, or by pacing the runner with a light motor-car carrying an automatic speed

TABLE XVIII.

SPEEDS OVER COURSES AT 1-KILOMETER AND 1-MILE DISTANCES.

I.	II.		III.	IV.	V.	VI.	VII.	VIII.	IX.	X.	XI.	XII.
Table.	Racers.		log *c*.	*c*.	Speed at 1 Kilometer.		Computed Time for 1 Kilometer, seconds.	Speed at 1 Mile.			Computed Time for 1 Mile, seconds.	Record Time for 1 Mile, seconds.
					log.	meters/seconds.		log.	meters/seconds.	miles/hours.		
II	Horses running		1.6274	42.4	1.2524	17.88	55.9	1.2266	16.85	37.70	95.5	95.5
III	Horses pacing		1.5363	34.38	1.1613	14.50	68.98	1.1355	13.66	30.56	117.8	115.25
I	Horses trotting		1.53	33.9	1.155	14.29	70.0	1.129	13.46	30.13	119.5	118.5
IX	Men skating		1.4143	25.96	1.0393	10.95	91.35	1.0135	10.31	23.08	156.0	156.0
IV	Men running		1.2307	17.01	0.8557	7.173	139.4	0.8299	6.759	15.12	238.1	252.75
VI	Men 8-oars	rowing	1.202	15.92	0.827	6.714	148.9	0.8012	6.327	14.15	254.3	304.75
VI	Men 4-oars		1.1145	13.02	0.7395	5.489	182.2	0.7137	5.173	11.57	311.1	291.0
VI	Men singles		1.085	12.16	0.710	5.129	195.0	0.6842	4.833	10.81	333.0	301.0
V	Men walking		1.0646	11.6	0.6896	4.893	204.4	0.6638	4.611	10.32	349.0	383.0
VIII	Men swimming		0.4196	2.628	0.0446	1.108	902.4	0.0188	1.044	2.336	1544.0	1476.2

recorder, or by noting on a chronograph the times of the runner's passage past a suitable number of fixed points along the track.

Although nothing can be stated directly from the data in this paper as to the degree of uniformity or of variation in the speed of a record-making trained racer, yet if it is proper to apply the inference drawn from a long series of complete races to the speed conditions during the operations of any one taken singly, then it should follow that the speed of a record-maker is very nearly uniform throughout the whole course. If, as appears from the whole series, the time of exhaustion varies inversely as the ninth power of the velocity, and this condition applies within the limits of any single race, then it is easily seen that the quickest way to reach the winning post is to take at the outset that speed which will just produce exhaustion at the goal, and keep to that speed throughout the course. The penalty for raising the speed at any part would be a degree of untimely exhaustion far outweighing the benefit gained. Trainers commonly direct practising athletes to spurt, or accelerate, near the end of the run. This advice must be sound on any theory, because to slacken speed at the end, if there is any balance of running energy left, would be absurd. The runner naturally expends all the available energy balance on the last lap; but if he is able to accelerate to any appreciable extent, it must mean that he has kept too much energy in reserve, and he would have done better to adopt a higher general speed. If, on the contrary, his pace falls to any appreciable extent at the end, he would have economized time by maintaining a lower general speed. Experimental evidence to test this theory would be of great interest. If the theory is correct, athletes, in training for a given event, ought to be motor-paced, the speed of the pacing motor being set uniform. In the earlier practice, this motor-speed should be, say, 15 per cent less than the desired record-speed, and the athlete should train to keep close to the motor. As the training progressed, the uniform speed of the motor over the course should be raised, say, 1 per cent at a time. Of course these suggestions advance beyond the warrant of evidence at this time.

CONCLUSIONS.

An analysis of the various national and international appended racing records, as above detailed, leads to the following conclusions, for trotting, pacing, and running horses, as well as for running, walking, rowing, skating, and swimming men:

(a) The time varies approximately as the ninth power of the eighth root of the distance. Doubling the distance means increasing the time 118 per cent (Table XIII).

(b) The time occupied in a record-making race varies approximately inversely as the ninth power of the speed over the course. Doubling the speed cuts down the racing time 512 times (Table XIV).

(c) The distance covered increases approximately as the eighth power of the ninth root of the time. Doubling the time of the race allows of increasing the course length by 85 per cent (Table XV).

(d) The distance covered increases approximately as the inverse eighth power of the speed over the course. Doubling the speed cuts down the distance that can be covered 256 times (Table XVI).

(e) The speed over the course varies approximately as the inverse eighth root of the distance. Doubling the distance brings down the speed about 9.3 per cent (Table XVII).

(f) The speed over the course varies approximately as the inverse ninth root of the racing time.

It may be noted that all of the statements (a) to (f) are different aspects of one and the same fact.

(g) If any of the three quantities L, T, and $V = \frac{L}{T}$ be plotted on logarithm paper as ordinates to either of the other quantities as abscissas, the record points will fall on, or near to, a straight line (Figures 2, 3, 6, 7, 8, 10, 11, 12, 13, and 15).

(h) Athletes aspiring to break racing records might succeed better in attacking those whose points fall below the straight lines of speed against distance, or above the straight lines of time against distance, rather than those whose points fall on the opposite sides of those lines.

(i) The records presented on bicycling do not determine the proper highest speed of cycling below 30 miles (48 kilometers), since there is apparently no reduction in speed by fatigue up to that distance.

(j) With the exception of bicycling, as above noted, the law of fatigue in racing is the same, or very nearly the same, with horses as with men, in air or in water, as indicated by the records analyzed in this paper.